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A TEST OF THE ALGEBRA OF BILOCAL OPERATORS

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ABSTRACT

We show that the electroproduction of massive muon pairs provides a test of the algebra of bilocal operators. The cross section for the contribution of the Compton term is derived and estimates of the background terms are indicated. As expected, numerical estimates indicate that the cross sections are fairly small.

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INTRODUCTION

During the last few years one of the most fruitful ideas in elementary particle physics has undoubtly been the algebraic scheme of equal time commutators for current densities. A natural extension of this idea has recently been proposed,[1] in which the commutators of current densities are postulated not only for equal times, but also for all light-like distances. The simplest of these schemes is abstracted from the free quark model. The densities are now bilocal operators made out of the quark fields and are given by:

$$J^1(x,y) = :\bar{\psi}(x)\Gamma^1\lambda^1\psi(y): \qquad (1.1)$$

where Γ^1 is a combination of γ matrices and λ^1 is a 3 x 3 unitary matrix. The new algebra postulates that when all possible separations are light-like:

$$(x-y)^2 = (u-v)^2 = (x-u)^2 = (y-u)^2 = (x-v)^2 = (y-v)^2 = 0 \qquad (1.2)$$

the commutation relations of the bilocal operators are those suggested by the free quark model:

$$\left[J^1(x,y), J^2(u,v)\right] \hat{=} \partial_\rho\Delta(x-v) \;:\bar{\psi}(u)\Gamma^2\gamma_\rho\Gamma^1\lambda^2\lambda^1\psi(y):$$
$$-\partial_\rho\Delta(y-u) \;:\bar{\psi}(x)\Gamma^1\gamma_\rho\Gamma^2\lambda^1\lambda^2\psi(v): \quad . \qquad (1.3)$$

The quark fields occuring in (1.1) and (1.3) are used symbolically in order to exhibit the SU(3) and Lorenz structure of the bilocal operators. The limited vilidity of the commutator is emphasized by the symbol

$\hat{=}$, denoting equality on the light cone.

Relations of this type have already been used successfully in order to explain the recent SLAC data on the deep inelastic electron-proton scattering.[1] In this case $J^1(x, y)$ and $J^2(u, v)$ are the ordinary local electromagnetic currents with $x=y$ and $u=v$. The implications of this scheme were also found to be in agreement with all the results of the quark-parton model,[2] which do not depend on the explicit assumptions about the momentum distribution of the partons. However, all these tests involve single commutators in which the initial densities are local operators. Recently, several difficult experiments have been proposed[3] in order to test the algebra of bilocal operators. In this paper we present an investigation for one of them, namely the electroproduction of massive muon pairs:

$$e + p \rightarrow e + (\mu^+ + \mu^-) + X \tag{1.4}$$

This test is of interest because it could measure, as we shall show, the connected part of the light-cone commutator of two retarded light-cone commutators.

To lowest order of electromagnetic interactions, the electroproduction of muon pairs involves three types of Feynman diagrams, which are shown in Fig. 1. We shall show that, in the appropriate kinematic regions, practically all the terms arising from them can be evaluated if one uses the algebra of Eq. (1.3). However, of main interest to this article is the square of diagram (1a), which provides a test for the algebra of bilocal operators. The other terms, although in principle known, should be regarded as background.

In Section II we exhibit clearly the assumptions under which the square of the Compton diagram (1a) can be evaluated using Eq. (1.3). Section III analyzes the contribution of the interference terms, using the new algebra, and indicates a set of experiments which isolate the incoherent sum of the three diagrams. Numerical results for the cross section arising from the Compton term are presented in Section IV, where we also summarize the main conclusions. One of the main difficulties in such experiments, however, arises from the large contribution of the background terms discussed in the appendix. Detailed numerical estimates of their contribution has not been performed, because of the great complexity of the formulae, but we indicate some analytic methods that could be used in estimating the backgrounds in a given experimental situation.

II. THE COMPTON PART OF THE AMPLITUDE

In this section we evaluate the contribution of the square of diagram (1a) to the cross section of process (1.4). The notation is shown in the figure. This diagram gives a term to the amplitude which can be written as:

$$F_1 = \frac{e^4}{k^2 q^2} T^{\mu\nu} \tau_{\mu\nu} \tag{2.1}$$

where $\tau_{\mu\nu}$ is the leptonic part given by:

$$\tau_{\mu\nu} = \left[\overline{U}_{(e)}(k_2)\gamma_\mu U_{(e)}(k_1)\right]\left[\overline{U}_{(\mu)}(q_1)\gamma_\nu V_{(\mu)}(q_2)\right] \tag{2.2}$$

and $T^{\mu\nu}$ is the off-mass shell "Compton" amplitude from an initial proton state of momentum p to an arbitrary, but fixed, final state $|n\rangle$ with momentum p'.

$$T^{\mu\nu} = \int d^4x\, e^{-iqx} \langle n | T^*\left(J^{\mu}(x),\ J^{\nu}(0)\right) | p\rangle \tag{2.3}$$

The matrix element in (2.3) represents the connected part of the covariant time-ordered product of two electromagnetic currents. The main objective is to find a kinematic region in which the light cone, $x^2 \sim 0$, dominates the integral of Eq. (2.3). We are thus led to examine the limit when both q_0 and q^2 becomes large but with fixed ratio. This limit is achieved when, in the laboratory frame, $k_0 \to \infty$, $q_0 = k_0 + \beta$, $q_3 = q_0 - \alpha$ while α, β and p' are being held finite. In this limit the exponential becomes $e^{iq_0(x_0 - x_3)} e^{i\alpha x_3}$ and a naive application of the stationary phase argument[4] suggests that the dominant contribution comes from $x^2 \sim \frac{1}{q_0}$. Therefore, we are tempted to apply the light cone expansion ideas to the T^* product of Eq. (2.3). However, this is not straightforward. In general, Eq. (2.3) might contain "sea-gull" terms together with non covariant operator Schwinger terms. This is, for example, the case when there is a fundamental charged scalar field in the theory. In this case, a light cone expansion similar to Eq. (1.3) might still be true for a given physical matrix element, but we do not know of any obvious way to see it. Therefore, rather than investigating the most general case, we limit ourselves to remark that the presence of fundamental charged scalar fields would have resulted in a longitudinal cross-section σ_L for the SLAC data, which does not vanish in the Bjorken limit. If we assume that this is not the case we can conjecture[1] that the behaviour of the currents near the light-cone is basically given by expressions containing quark fields like in Eq. (1.1). In this case the above mentioned complications are hopefully absent. Therefore, we feel confident to assume that, near the light cone $x^2 \sim 0$, the T^* product of Eq. (2.3) can be replaced by an ordinary commutator of two quark-current densities multiplied by a step function. Thus, using Eq. (1.3), we can write:

$$T^{\mu\nu} \mathrel{\hat{=}} \int d^4 x\, e^{-iqx} \theta(x^\circ) \partial_\rho \Delta(x)$$

$$\langle n | \left(S^{\mu\nu\rho\lambda} A_\lambda(0,x) - \epsilon^{\mu\nu\rho\lambda} S_{\lambda 5}(0,x) \right) | p \rangle \tag{2.4}$$

where

$$S^{\mu\nu\rho\lambda} = g^{\mu\rho} g^{\nu\lambda} + g^{\mu\lambda} g^{\nu\rho} - g^{\mu\nu} g^{\rho\lambda} \tag{2.5}$$

$$A^\lambda(x,0) = J^\lambda(x,0) - J^\lambda(0,x) \tag{2.6}$$

$$S_5^\lambda(x,0) = J_5^\lambda(x,0) + J_5^\lambda(0,x) \tag{2.7}$$

The SU(3) content of the currents $J^\lambda(x,0)$ and $J_5^\lambda(x,0)$ is shown if we write them, symbolically, in terms of quark fields:

$$J^{\lambda}(x,0) = \frac{1}{2} :\bar{\psi}(x)\gamma^{\lambda}Q^2\psi(0): \ , J_5^{\lambda}(x,0) = \frac{1}{2} :\bar{\psi}(x)\gamma^{\lambda}\gamma_5 Q^2\psi(0): \tag{2.8}$$

Q is the charge matrix: $Q = \begin{pmatrix} 2/3 & & 0 \\ & -1/3 & \\ 0 & & -1/3 \end{pmatrix}$

In order to calculate the cross-section we square this term:

$$|F_1|^2 = \frac{e^8}{q^4 k^4} T^{\mu\nu} T^{*\alpha\beta} \ \mathcal{T}_{\mu\nu} \mathcal{T}^*_{\alpha\beta} \tag{2.9}$$

The leptonic part of (2.9), summed over lepton spins and integrated over the relative momentum of the two final muons, gives:

$$\sum \mathcal{T}_{\mu\nu} \mathcal{T}^*_{\alpha\beta} = \frac{\pi}{6} q^2 g_{\mu\alpha} \left[k_\nu k_\beta - \mathcal{T}_\nu \mathcal{T}_\beta - k^2 g_{\nu\beta} \right] \tag{2.10}$$

where $\mathcal{T} = k_1 + k_2$ and the gauge invariance of the hadronic part has been used. Notice that, as it is written, (2.10) satisfies the gauge conditions:

$$k^\nu \sum \mathcal{T}_{\mu\nu} \mathcal{T}^*_{\alpha\beta} = k^\beta \sum \mathcal{T}_{\mu\nu} \mathcal{T}^*_{\alpha\beta} = 0 \tag{2.11}$$

Let us now look at the hadronic part. We square Eq. (2.4) and sum over the spin of the proton and the intermediate states $|n>$. Taking into accound (2.10) we obtain:

$$\sum_{\substack{\text{spin p.} \\ \text{states n.}}} T^{\mu\nu} T^{*\beta}_{\mu} (2\pi)^4 \delta^4(k+p-q-p') \hat{=}$$

$$\int d^4x\, d^4y\, d^4z\, e^{iq(y+z-x)} e^{-ikz} \theta(x^\circ)\theta(y^\circ)\partial_\rho \Delta(x)\partial_\tau \Delta(y)$$

$$<p\left| \left[\left(S_\mu^{\ \beta\tau\gamma} A_\gamma(y+z,z) + i\ \epsilon_\mu^{\ \beta\tau\gamma} S_{\gamma,5}(y+z,z) \right), \right.\right.$$

$$\left.\left. \left(S^{\mu\nu\rho\lambda} A_\lambda(0,x) - i\epsilon^{\mu\nu\rho\lambda} S_{\lambda,5}(0,x) \right) \right] \right| p> \tag{2.12}$$

In writing Eq. (2.12) the usual argument about the energy of the intermediate states has been used in order to convert the product of the bilocal operators into a commutator. Note that $x^2 \sim y^2 \sim 0$. We want to use again a stationary phase argument in order to conjecture that the integral in (2.12) is dominated by the light cone with respect to all pairs of points. In the first limit we had taken $(k_0+q_0) \to \infty$ keeping k_0-q_0 finite. Now we take k_0-q_0 large in the same way, namely $k_0-q_0 = \beta \to \infty$ keeping $\alpha = q_0 - q_3$ finite. The order of the two limiting procedures is very important. It is more convenient to express these limits in terms of scalar quantities. Let us define:

$$S = (k+p)^2, \qquad t = (k-q)^2, \qquad 2M\nu = 2p\cdot(k-q) \tag{2.13}$$

The first limit consisted in taking k^2, q^2 and S large with fixed ratios, keeping t and $2M\nu$ finite. In the second step we take also the limit $-t$, $2M\nu \to \infty$ keeping the ratio $\xi = -t/2M\nu$ finite. This order of limits corresponds to a well-defined "gedanken" experiment but we hope that it can also describe the situation where k^2, q^2, $S \gg -t$, $2M\nu \gg M^2$. In this limit it follows that $(z_0 - z_3)^2 \sim 0$ and causality implies[5] that all six differences are also light-like. Therefore, we can use the commutators of Eq. (1.3) and taking into accound the symmetry of the leptonic part in $\beta \leftrightarrow \nu$ we obtain:

$$\sum T^{\mu\nu} T^{*}{}_{\mu}{}^{\beta} (2\pi)^4 \delta^4(k+p-q-p') =$$

$$S^{\gamma\lambda\sigma k}\left(S_{\mu}{}^{\beta\tau}{}_{\gamma}\, S^{\mu\nu\rho}{}_{\lambda} - \epsilon_{\mu}{}^{\beta\tau}{}_{\gamma}\, \epsilon^{\mu\nu\rho}{}_{\lambda}\right)\left(R^{(1)}_{\rho\tau\sigma k} + R^{(2)}_{\rho\tau\sigma k}\right)$$

$$- S^{\gamma\lambda\sigma k}\left(S_{\mu}{}^{\beta\tau}{}_{\gamma}\, S^{\mu\nu\rho}{}_{\lambda} + \epsilon_{\mu}{}^{\beta\tau}{}_{\gamma}\, \epsilon^{\mu\nu\rho}{}_{\lambda}\right)\left(R^{(3)}_{\rho\tau\sigma k} + R^{(4)}_{\rho\tau\sigma k}\right)$$

$$+ 2\, S_{\mu}{}^{\beta\tau}{}_{\gamma}\, \epsilon^{\mu\nu\rho}{}_{\lambda}\, \epsilon^{\gamma\lambda\sigma k}\left(R^{(1)}_{\rho\tau\sigma k} - R^{(2)}_{\rho\tau\sigma k} + R^{(3)}_{\rho\tau\sigma k} - R^{(4)}_{\rho\tau\sigma k}\right) \qquad (2.14)$$

where the R's are given by:

$$R^{(1)}_{\rho\tau\sigma k} = \frac{\pi}{2}\, \frac{N_\rho L_\tau R_\sigma P_k}{N^2 L^2}\, \frac{A_{Q^4}(\xi)}{2M\nu} \qquad (2.15)$$

$$R^{(2)}_{\rho\tau\sigma k} = R^{(1)}_{\tau\rho\sigma k} \qquad (2.16)$$

$$R^{(3)}_{\rho\tau\sigma k} = \frac{\pi}{2}\, \frac{L_\rho L_\tau R_\sigma P_k}{L^4}\, \frac{A_{Q^4}(\xi)}{2M\nu} \qquad (2.17)$$

$$R^{(4)}_{\rho\tau\sigma k} = \frac{\pi}{2}\, \frac{N_\rho N_\tau R_\sigma P_k}{N^4}\, \frac{A_{Q^4}(\xi)}{2M\nu} \qquad (2.18)$$

$$N = q - \xi p, \quad L = k + \xi p, \quad R = k - q + \xi p \qquad (2.19)$$

$$\langle p \,|\, A_k(u,v) \,|\, p\rangle = 2p_k\, \widetilde{A}_{Q^4}\left(p\cdot(u-v)\right) = 2p_k \int_{-\infty}^{\infty} e^{-i\eta\, p\cdot(u-v)} A_{Q^4}(\eta)\, d\eta \qquad (2.20)$$

and $A_k(u,v)$ in Eq. (2.20) is defined in exactly the same way as in Eq. (2.6, 2.7) but the currents $J_\mu(u,v)$ contain now the matrix Q^4, instead of Q^2. Therefore, the SU(3) decomposition of $A_{Q^4}(\xi)$ is:

$$A_{Q^4}(\xi) = \frac{4}{27}\, A^{(0)}(\xi) + \frac{5}{27}\, A^{(3)}(\xi) + \frac{5}{27\sqrt{3}}\, A^{(8)}(\xi) \qquad (2.21)$$

The functions $A^{(0)}$, $A^{(3)}$ and $A^{(8)}$ are in turn given by the electro- and neutrino production data[1] by:

$$A^{(0)}(\xi) = \frac{3}{2\xi}\left[3\left(F_2^{ep} + F_2^{en}\right) - 1/2\left(F_2^{\nu p} + F_2^{\nu n}\right)\right] \tag{2.22}$$

$$A^{(3)}(\xi) = \frac{3}{\xi}\left[F_2^{ep} - F_2^{en}\right] \tag{2.23}$$

$$A^{(8)}(\xi) = \frac{\sqrt{3}}{\xi}\left[(F_2^{\nu p} + F_2^{\nu n}) - 3(F_2^{ep} + F_2^{en})\right] \tag{2.24}$$

In particular, the difference of the Compton contribution to experiments with proton and neutron targets is completely determined by the electroproduction data:

$$A^p_{Q^4} - A^n_{Q^4} = \frac{5}{9}\left(A^p_{Q^2} - A^n_{Q^2}\right). \tag{2.25}$$

Finally, the ratio A_{Q^4}/A_{Q^2} can be bounded

$$1/9 \le A_{Q^4}/A_{Q^2} \le 4/9 \text{ (for a proton target)} \tag{2.26}$$

by using the structure function inequalities[6] that have been obtained for electroproduction.

Putting together equations (2.11) and (2.14), we obtain, after some algebra, the final form of the hadronic tensor:

$$\sum T^{\mu\nu} T^{*}{}_{\mu}{}^{\beta} (2\pi)^4 \delta^4(p+k-q-p') = \frac{\pi A_{Q^4}(\xi)}{2M\nu} \Bigg\{$$

$$g^{\beta\nu}\left[-\frac{2(N\cdot R)(N\cdot P)}{N^4} - \frac{2(L\cdot R)(L\cdot P)}{L^4} - 2M\nu\frac{(L\cdot N)}{L^2N^2} + M\nu\left(\frac{1}{L^2} + \frac{1}{N^2}\right) + \frac{2}{L^2N^2}\left((P\cdot N)(L\cdot R) - (P\cdot L)(N\cdot R)\right)\right]$$

$$-N^{\nu}N^{\beta}\left[\frac{4\cdot(P\cdot N)}{N^4} + \frac{4\,(P\cdot L)}{L^2N^2}\right]$$

$$+ p^{\nu}N^{\beta}\left[2\left(\frac{1}{L^2}+\frac{1}{N^2}\right)+4\xi\frac{(P\cdot N)}{L^2N^2}-4\frac{(L\cdot R)}{L^2N^2}\right]$$

$$+ p^{\nu}p^{\beta}4\xi\left[\frac{(L\cdot R)}{L^4}+\frac{(N R)}{L^2N^2}\right]\Bigg\} \tag{2.27}$$

The cross section can now be obtained trivially by multiplying (2.27) by the electron tensor. The explicit formula for the cross section is

$$\frac{d\sigma}{dq^2 dq_0 dk_2 d\Omega_{k_2}} = \frac{\alpha^4}{12\pi^2}\left(\frac{k_2}{k_1}\right)\frac{\left(q_0^{\,2}-q^2\right)^{1/2}}{k^4q^2}\;\frac{A_Q 4(\xi)}{2M^2\nu}$$

$$\Bigg\{k^2\left[4\frac{(N\cdot R)(N\cdot p)}{N^4}+4\frac{(L\cdot R)(L\cdot p)}{L^4}-2M\nu\left(\frac{1}{L^2}+\frac{1}{N^2}\right)+4M\nu\frac{L\cdot N}{L^2N^2}\right.$$

$$\left.-\,{}^4\!/_{L^2N^2}\Big((N\cdot p)(L\cdot R)-(L\cdot p)(L\cdot R)\Big)\right] \tag{2.28}$$

$$-4\left[(k\cdot N)^2-(\tau\cdot N)^2-k^2N^2\right]\left[\frac{p\cdot N}{N^4}+\frac{p\cdot L}{N^2L^2}\right]+4\xi\left[(k\cdot p)^2-(\tau\cdot p)^2-M^2k^2\right]\left[\frac{L\cdot R}{L^4}+\frac{N\cdot R}{L^2N^2}\right]$$

$$+\left[(k\cdot p)(k\cdot N)-(\tau\cdot p)(\tau\cdot N)-p\cdot N k^2\right]\left[2\left(\frac{1}{L^2}+\frac{1}{N^2}\right)+4\xi\frac{p\cdot N}{L^2N^2}-4\frac{L\cdot R}{L^2N^2}\right]\Bigg\}$$

III. INTERFERENCE TERMS

The same analysis can be extended to include the interference between diagram (1a) and the rest of the diagrams. The hadronic tensor is given by

$T^{\mu\nu}T^{\rho *}$ + c. c., where

$$T^{\mu\nu}T^{\rho *}\delta^4(k+p-q-p') = \int dydz\, e^{iqy} e^{i(k-q)z}\, \theta(y_o) < p \left| \left[\left[J^\mu(y+z),\ J^\nu(z) \right],\ J^\rho(0) \right] \right| p>. \tag{3.1}$$

Repeating the arguments of the previous section we obtain that the relevant distances are again on the light cone. The main difference is that, in the scaling limit, the interference terms are proportional to

$$<p \left| \left[A^\lambda(o,z) + S_5^\lambda(o,z),\ J^\alpha(y) \right] \right| p>. \tag{3.2}$$

The terms that survive now are of the form $<p | S^k(x,y) | p>$ and not $<p | A^k(x,y) | p>$. Therefore, we need the symmetric function $S^{(0)}(\xi)$ and $S^{(8)}(\xi)$. In fact $S^k(x,y)$ contains the matrix Q^3, due to the three electromagnetic currents, which has the SU(3) decomposition:

$$S_{Q^3}(\xi) = \frac{4}{27}\, S^{(0)}(\xi) + \frac{1}{3}\, S^{(3)}(\xi) + \frac{1}{3\sqrt{3}}\, S^{(8)}(\xi) \tag{3.3}$$

From neutrino data we know $S^3(\xi)$ because it is proportional to $F_2^{\nu p} - F_2^{\nu n}$, and the combination $\frac{2}{3}S^{(0)}(\xi) + \frac{1}{\sqrt{3}}\, S^{(8)}(\xi)$ because it is proportional to $F_3^{\nu p} + F_3^{\nu n}$. But we do not know $S^{(0)}(\xi)$ and $S^{(8)}(\xi)$

separately. Therefore this term, although interesting from a theoretical point of view, cannot be evaluated using electro- and neutrino-production data.

Fortunately there are experiments where the interference terms drop out. By integrating over the relative phase space of the muon pair, the interference between diagrams (1c), (1c′) and the diagrams (1a), (1b), (1b′) can be eliminated. In addition, by doing experiments with electrons and positrons and summing up the cross sections we can eliminate the interference between (1a) and (1b), (1b′). Thus we can obtain the incoherent sum of the three different types of diagrams. Alternatively, the difference of the cross sections for incident electrons and positrons measures directly the symmetric function $S_{Q^3}(\xi)$. The same term can also be separated in electroproduction experiments by selecting that part of the cross section, which is antisymmetric in the muons.

IV. NUMERICAL RESULTS AND CONCLUSIONS

We consider the experimental situation of an incident electron beam with momentum k_1 on a proton target leading to a final electron with momentum k_2, a dimuon pair and an unobserved hadronic state. The momentum transfer between the electrons is indicated by k^2.

In the dimuon system we integrate over their relative phase-space, thus characterizing the system by a single four vector q_μ. In addition to the five variables characterizing the hadronic part of the process, we have two additional variables characterizing the leptons. One is the ratio of the electron energies k_2/k_1. The other variable is the angular correlation ϕ, between the normal to the electron plane and the vector $\vec{q}$. An averaging over the angles ϕ has been performed in the numerical values of the cross section given in Table 1. In these numerical estimates we also set $A_{Q^4}(\xi) = A_{Q^2}(\xi)$. As it is seen from the table, the dependence of the cross section of the variables k^2, q^2 is fairly rapid, so that smaller values for these variables lead to considerably larger cross sections.

A final consideration is the contribution of the other diagrams. We have calculated in the appendix the hadronic tensor for the bremsstrahlung diagram and we also obtained a lower bound for the contribution of the Bethe-Heitler terms. The contributions from the Bethe-Heitler terms can be, in our specific limits, fairly large because of the $1/t$ dependence occuring in Equation (A-14).

In summarizing the results, we note that the electroproduction of massive muon pairs provides a theoretical laboratory in the neighborhood of the light-cone; since it involves not only the commutator of two local operators, but also the commutators of bilocals with bilocals, as well as the commutator of locals with bilocals. The picture that emerges from our analysis indicates that the cross sections calculated in specific kinematic regions, where light-cone techniques can be applied, are completely predictable in terms of the electro- and neutrino-production data. The cross sections are small and the backgrounds formidable, but perhaps not unmanageable.

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APPENDIX

Background Terms

What we call here "background terms" are represented by the four diagrams of Fig. (1b), (1b'), (1c) and (1c'). They represent: (i) The "bremsctrahlung" part of Fig. (1b) and (1b') whose contribution to the amplitude is

$$F_2 = \frac{e^4}{q^2(k-q)^2} T^\mu \sigma_\mu \tag{A-1}$$

where the notation is defined on Fig. 1 and

$$T^\mu = \langle n| J^\mu(0) |p\rangle \tag{A-2}$$

$$\sigma_\mu = \left[\overline{U}_{(e)}(k_2)\left(\gamma^\nu \frac{1}{\not q+\not k_2}\gamma_\mu + \gamma_\mu \frac{1}{\not k_1-\not q}\gamma^\nu\right)U_{(e)}(k_1)\right]\left[\overline{U}_{(\mu)}(q_1)\gamma_\nu V_{(\mu)}(q_2)\right] \tag{A-3}$$

(ii) The Bethe-Heitler terms of Fig. (1c) and (1c') given by:

$$F_3 = \frac{e^4}{k^2(k-q)^2} T^\mu \chi_\mu \tag{A-4}$$

with

$$\chi_\mu = \left[\overline{U}_{(e)}(k_2)\gamma_\nu U_{(e)}(k_1)\right]\left[\overline{U}_{(\mu)}(q_1)\left(\gamma^\nu \frac{1}{\not k-\not q_1}\gamma_\mu + \gamma_\mu \frac{1}{\not k-\not q_2}\gamma^\nu\right)V_{(\mu)}(q_2)\right]$$

$$= \overline{U}_{(e)}(k_2)\gamma_\nu U_{(e)}(k_1)\, M^\nu{}_\mu \ . \tag{A-5}$$

Taking the square of $F_2 + F_3$ we obtain:

$$|F_2+F_3|^2 = \frac{e^8}{(k-q)^4} T^\mu T^{\nu *}\left(\frac{\sigma_\mu}{q^2}+\frac{\chi_\mu}{k^2}\right)\left(\frac{\sigma_\nu^*}{q^2}+\frac{\chi_\nu^*}{k^2}\right) \tag{A-6}$$

The hadronic part, summed over the spin of the proton and the intermediate states $|n>$ is easy to evaluate. Indeed we have:

$$\sum_{\substack{\text{Spin } p \\ \text{State } n}} T^{\mu} T^{\nu *} (2\pi)^4 \delta^4(k+p-q-p') =$$

$$\int <p| \left[J^{\mu}(x), J^{\nu}(0) \right] |p> e^{i(q-k)x} d^4x \qquad \text{(A-7)}$$

In the limit $t \to -\infty$, $2M\nu \to \infty$ with fixed ξ the integral in (34) is dominated by the light cone and the matrix element gives just the ordinary electroproduction structure function:

$$\sum T^{\mu} T^{\nu *} (2\pi)^4 \delta^4(k+p-q-p') = - \frac{\pi\, p_\lambda R_\rho\, S^{\mu\nu\lambda\rho}}{\nu M} A_{Q^2}(\xi) \qquad \text{(A-8)}$$

$$A_{Q^2}(\xi) = \frac{4}{9} A^{(0)}(\xi) + \frac{1}{3} A^{(3)}(\xi) + \frac{1}{3\sqrt{3}} A^{(8)}(\xi) \qquad \text{(A-9)}$$

As we see, the contribution of F_2 and F_3, although completely determined in the kinematical region we are interested, does not contain the light-cone commutator of bilocal operators. Therefore, we consider them as background to the term F_1 which alone gives a test of the full algebra of Eq. (1.3).

Let us now turn to the leptonic part. Because the muon pairs in F_2 and F_3 have opposite values of C, the interference between these two terms vanishes when integrating over the relative momentum of the muons. Therefore, we need only to calculate $\frac{1}{q^4} \sigma_\mu \sigma_\nu^* + \frac{1}{k^4} \chi_\mu \chi_\nu^*$.

After a rather lengthy calculation we find:

$$\sum \int |F_2|^2 = \frac{e^8}{q^2 t^2} \frac{\pi^2}{3} \frac{S^{\mu\nu\rho\lambda} p_\lambda R_\rho}{\nu M} A_{Q^2}(\xi) W_{\mu\nu} \tag{A-10}$$

where the summation is over the proton and lepton spins and the intermediate states n and the integration is over the relative momentum of the two muons. $W_{\mu\nu}$ is defined by:

$$\begin{aligned} W_{\mu\nu} = & \frac{4}{(k_1-s)^2(k_2+s)^2} \Big\{ -\frac{1}{2} g_{\mu\nu} t k^2 + 2 k_{1\mu} k_{2\nu} (k \cdot q) \\ & - 2 k_{2\mu} k_{2\nu} (k_1 \cdot s) + 2 k_{1\mu} k_{2\nu} (k_2 \cdot s) \Big\} \\ & + \frac{4}{(k_1-s)^4} \Big\{ g_{\mu\nu} \left[(k_1 \cdot s)(k_2 \cdot s) + \frac{1}{4} t k^2 \right] \\ & + k_{1\mu} k_{2\nu} (k_1 - s)^2 - 2 k_{1\mu} k_{1\nu} (k_2 \cdot q) \Big\} \\ & + \frac{4}{(k_2+s)^2} \Big\{ g_{\mu\nu} \left[(k_1 \cdot s)(k_2 \cdot s) + \frac{1}{4} t k^2 \right] \\ & + k_{1\mu} k_{2\nu} (k_2 + s)^2 + 2 k_{2\mu} k_{2\nu} (k_1 \cdot q) \Big\} \end{aligned} \tag{A-11}$$

where $s = k-q$.

For the contribution of the Bethe-Heitler term we obtain a lower bound. The lepton tensor corresponding to the electron vertex can be written as:

$$\tau_{\mu\nu} = \alpha\, \epsilon^S_\mu \epsilon^S_\nu + \beta \left(\epsilon^{*R}_\mu \epsilon^R_\nu + \epsilon^{*L}_\mu \epsilon^L_\nu \right) \tag{A-12}$$

where α and β are positive scalars and ϵ^S_μ, ϵ^R_μ, ϵ^L_ν are the scalar, right-handed and left-handed polarization vectors, respectively. The lower bound is obtained by replacing $\tau_{\mu\nu}$ by

$$\tau'_{\mu\nu} = \beta\left(\epsilon^{*R}_\mu \epsilon^R_\nu + \epsilon^{*L}_\mu \epsilon^L_\nu - \epsilon^S_\mu \epsilon^S_\nu\right)$$

$$= -\beta\left(g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right) \qquad \text{(A-13)}$$

The difference of the two tensors is the term $(\alpha+\beta)\,\epsilon^S_\mu \epsilon^S_\nu$, whose contribution to the cross section is positive. The cross section can now be bounded

$$\frac{d\sigma}{d^4q\, d^3k_2} \geq \frac{\alpha^4}{4\pi^5}\,\frac{1}{k_1 q^4 t}\left\{-W_1\left(3f_1+f_2\right) + W_2 f_1\left(1 - \frac{p\cdot s}{tM^2}\right)\right.$$

$$\left. + \frac{W_2 f_2}{k^2 - \frac{(k\cdot s)^2}{t}}\left(k\cdot p - \frac{(k\cdot s)(k\cdot p)}{t}\right)\right\} \qquad \text{(A-14)}$$

where

$$f_1 = \frac{1}{2}\left[L^\mu{}_\mu - \frac{L^{\mu\nu} k_\mu k_\nu}{k^2 - \frac{(k\cdot s)^2}{t}}\right];\quad f_2 = \frac{1}{2}\left[-L^\mu{}_\mu + 3\,\frac{L^{\mu\nu} k_\mu k_\nu}{k^2 - \frac{(k\cdot s)^2}{t}}\right] \qquad \text{(A-15)}$$

$$L_{\mu\nu} = \int \mathrm{Tr.}\left(M_{\lambda\mu} M^\lambda{}_\nu\right)\delta^4\left(k-s-q_1-q_2\right)\frac{d^3q_1}{E_{q_1}}\,\frac{d^3q_2}{E_{q_2}} \qquad \text{(A-16)}$$

W_1, W_2 are the usual structure functions and $M_{\mu\nu}$ is defined in Eq. (A-5). The traces occuring (A-16) are now fairly simple and the integrations can be performed analytically.

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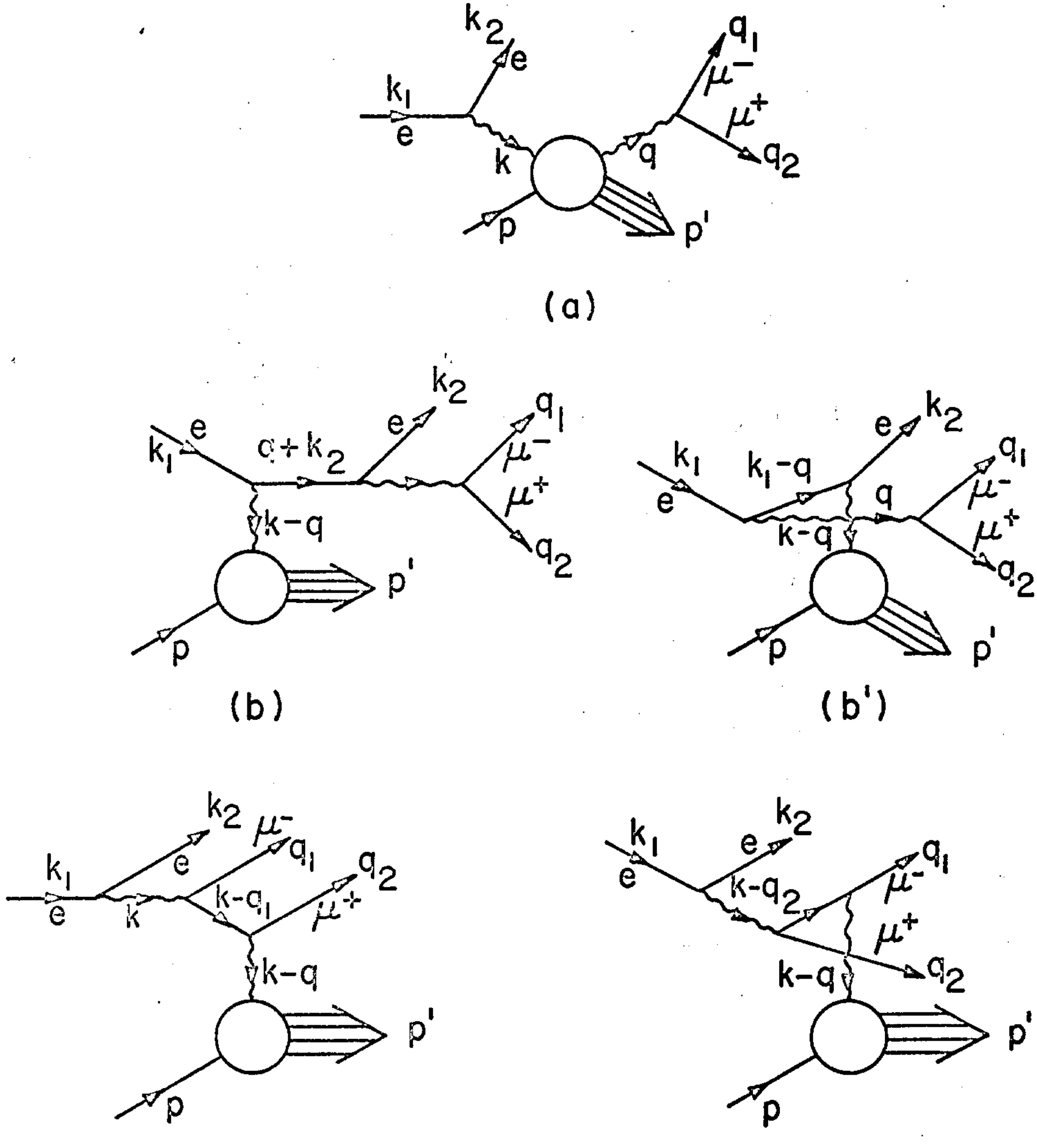
TABLE CAPTION

The Cross Section As A Function Of The Kinematic Variables. The specific values of the variables were chosen to satisfy the limits described in the text. There is no reason to attach any other significance to these particular values, since they were chosen randomley.

FIGURE CAPTION

Feynmann Diagrams Contributing To The Electroproduction Of Massive Muon Pairs.

ν in GeV	t in $(\mathrm{GeV})^2$	$\frac{d\sigma}{dq^2 dq_0 dk_2 d\Omega_2}$ in $\frac{\mathrm{cm}^2}{(\mathrm{GeV})^4(\mathrm{Str})}$	with
10.00	-5.54	$.26\times10^{-39}$	$k_1 = 80.00$ GeV, $k_2/k_1 = 3/8$, $q^2 = -k^2 = 12.00\ (\mathrm{GeV})^2$
8.00	-4.33	$.28\times10^{-39}$	
6.00	-3.19	$.29\times10^{-39}$	
4.00	-2.12	$.30\times10^{-39}$	
6.00	-1.50	$.49\times10^{-38}$	$k_1 = 80.00$ GeV, $k_2/k_1 = 3/8$, $q^2 = -k^2 = 6.00\ (\mathrm{GeV})^2$
4.00	-1.00	$.44\times10^{-38}$	


k2
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