CORRELATIONS IN HIGH-ENERGY PRODUCTION PROCESSES [†]

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ABSTRACT

Rapidity distributions for two particle inclusive reactions are calculated from the Nova model which has been successful at reproducing the single particle distributions. Particular attention is given to the $2\pi^-$ correlations in $\pi^\pm p$ and pp collisions for which data are available. Good agreement with data provides further evidence for a strong clustering of secondaries in particle production at present machine energies, a feature inferred previously from the analysis of single particle distributions. We stress the importance of measuring the energy dependence of $<n(n-1)>$, the quantity which normalizes the two particle distribution. A number of other features of correlation experiments are discussed. General properties of particle production are emphasized in reviewing the Nova model.

I. Introduction.

Inelastic processes represent approximately 80% of the total strong interaction cross section at high energy. However, the large number of potentially important kinematic variables necessary for a complete description of these processes has hampered progress toward their understanding. Finding good ways to display available data is a fascinating challenge. Advent of the inclusive approach[1-4] has simplified matters somewhat. Single particle inclusive cross sections may be described by only three kinematic variables, but even here, a particular choice of a given set must be made. These complications make it hard to approach particle production without some prejudices, even though one would wish to remain open to all suggestions obtainable from an unbiased contemplation of data. Fortunately, a motivated interest in one or a few variables may be rewarded by some insight and suggest further investigations. This has certainly been the case with the single-particle distributions, where rapidity y and transverse momentum squared p_T^2 distributions of one specific secondary are measured[5], with everything else summed or averaged over. Single-particle distributions are quite accessible experimentally, and they appear to provide tests of asymptotically expected scaling properties.[1,2] Asymptotic limits and the approach to scaling have both been confronted in Regge theory[6], and

its associated phenomenology.[7] The next phase of inclusive phenome-
nology is a study of two-particle inclusive distributions and possible
correlations among produced secondaries.

A number of features of single-particle inclusive spectra encourage
the attempt to identify some dominant production mechanisms, which
could serve as a basis for analyzing reactions in more detail.[4]
Focusing on specific production mechanisms could be useful for the
complicated problem of two-particle correlations. Although such an
approach risks loss of generality, it is clear that detailed calculations
are necessary if progress is to be made. General arguments based
on the use of sum rules[8] have been developed, but their implications
for data are as yet very weak, in particular for slow pions in the center
of mass, where the cross section is largest. Energy-momentum
conservation <u>alone</u> imposes little constraint on shapes of single and
two-particle distributions. Because data on two-particle distributions
are accumulating rapidly;[(9-12)] is should be extremely useful to have
explicit model predicitions from which to view the data.

Asymptotic predictions for two-particle correlations have already
been studied in the framework of Regge, dual, and diffraction
models.[(13-16)] In these models correlations are expected to decrease
with increasing rapidity difference ($\Delta y \geq 2$), but some models require
a very large center of mass energy before detailed study is possible.

This is especially true of models which postulate relatively weak correlations among secondaries originating from different clusters along a multi-exchange chain. One may also use the Feynman gas analogy[1, 16] applicable when the rapidity distribution has become asymptotically uniform. In the gas, clustering fluctuations may occur which will lead to short-range correlations, and a definite correlation length in Δy. A problem in applying this analogy to present machine energies is that in several interesting cases the number of "gas molecules" involved, on the average, equals one to two (See Table I). In spite of the value of asymptotic approaches and formal analogies, presently available data must also be studied along different lines.

A glance at the single-pion distributions already provides some model independent expectations for two-pion distributions. Figure 1 shows the π^- distributions in 18.5 GeV/c π^+p and π^-p collisions.[9] The shape in rapidity is narrow and Gaussian-like. In addition a strong leading particle effect is present in the π^-p reaction. Whatever the reason for the Gaussian-like shape, the narrowness of the rapidity distribution indicates that the two-particle distribution will have its maximum when both rapidities are small, and hence, when the rapidity difference is small. The experimental evidence is for a prominent maximum at zero rapidity difference in the two π^- system.[10, 11] The double distribution[9, 12] $d^2\sigma/dy_1 dy_2$ peaks near $y_1 = 0$ for any fixed y_2 with the maximum shifting only lightly as a function of y_2.

Thus $d^2\sigma/dy_1 dy_2$ displays much more information[9,12] than $d\sigma/d(\Delta y)$ which shows only a deceptive maximum near $\Delta y = 0$. A very interesting problem is to account for the narrowness of both the one- and two-pion distributions.

The observed strength and distribution of the leading particle effects in πp and pp collisions indeed provide some understanding why the single-pion rapidity distributions are so narrow.[17,18] In many events, one of the initial particles is quasielastically scattered and carries off a sizable fraction of the energy.[19] The energy left to be shared among the remaining secondaries is small enough that produced pions tend to cluster at small rapidities. This is the weak inelasticity property which dominates production processes at present machine energy.[20] It finds a rationale in several theoretical approaches. Moreover, the rapid scaling of the invariant pion distributions at intermediate values of $|x|$ strongly suggests that pions which reach these intermediate values of $|x|$ come from energy independent production of low mass clusters. Very little more than this is needed for understanding single-particle distributions and, as discussed here, the correlation data. Since the mass of a cluster which gives two π^- must be relatively large, one would expect $d^2\sigma/dy_1 dy_2$, to be narrower than $d\sigma/dy$. It is possible to construct a rather general model based on these features because the distribution of secondaries from each cluster is quite independent of reaction and particle-type,

a feature suggested by the nearly universal p_T^2 distributions observed experimentally. We emphasize that the general behavior of the correlations can be accounted for by these experimental features of particle production. With only a few more assumptions they are easily incorporated into a specific model such as the Nova model. [21]

The object of this paper is to show that a model which incorporates the key features already mentioned and thus reproduces the single-particle distributions very well, [21] can also account for the two-particle distributions. [12, 22] The shapes are basically those expected from the observed small inelasticity, the interpretation of the rapid scaling at intermediate $|x|$ found in the model, and the general behavior of the p_T^2 distributions. [3, 4] The very general clustering effect already inferred from the analysis of single distribution is enough to explain the two-particle correlation results now available. Two-particle data therefore provide further tests for some ideas which were less well probed by the single-particle distributions, expecially some of the details of particle production, but do not suggest specific additional effects.

In this paper, we also emphasize the great value of obtaining experimental information on the energy dependence of a quantity which actually normalizes the two-particle distribution, namely $<n(n-1)>$. The analogous quantity in the single-particle case is the mean multiplicity, $<n>$, of the observed particle type.

$$\langle n(n-1)\rangle \, \sigma_{inel} = \int \frac{d^2\sigma}{dy_1 dy_2} \, dy_1 \, dy_2 \tag{1.1}$$

$$\langle n \rangle \, \sigma_{inel} = \int \frac{d\sigma}{dy} \, dy \tag{1.2}$$

One of the most revealing and sensitive quantities which model calculations must account for is the energy dependence of the ratio, $\langle n(n-1)\rangle / \langle n \rangle^2$ for secondaries of a specific kind.[23]

Calculations of $\pi^- \pi^-$ distributions are described in Sec. II. Strong correlations, already suggested by the single-particle distributions and observed in analyses of exclusive channels, are realized by associating most of the time the two observed pions with the excitations of either the beam or target particle into a resonance-like state (Nova) which decays subsequently by pion emission. The Nova model[21] is reviewed briefly in the course of setting up the two-particle inclusive calculation. Specific predictions of the model agree well with available correlation data. Calculations of the $\pi^+ p \ (\pi^-, \pi^-)$ distribution[24] at 18.5 GeV/c which have already been reported elsewhere,[22] are discussed in more detail. Those results showed deviations from data for large differences of rapidity where cross sections are small. This discrepancy suggests an obvious double excitation correction to the dominant single-excitation picture. We find that data are well accounted for if 70% of the inelastic cross section is associated with

single excitation, and the remainder comes from events in which both beam and target particles form clusters.[*] The 7/3 ratio was also found[12] to lead to a good representation of $pp(\pi^-, \pi^-)$. The ratio of single to double excitation is tested by the reaction, $\pi^- p(\pi^-, \pi^-)$ where $d^2\sigma/dy_1 dy_2$ is extremely sensitive to the presence of leading π^-'s. Results are presented here for $pp(\pi^-, \pi^-)^{(12)}$ and for $\pi^\pm p(\pi^-, \pi^-)$.[9]

Normalizing to σ_{inel}, we find that our model reproduces correctly the normalization of the two-particle distributions. Results of our calculations are compared with data in Fig. 1, 3, 4, 5, and 6. The agreement with experiment provides evidence that the pion secondaries are indeed strongly correlated at present machine energies, but in a well understood way. Correlations from final state interactions are minimized by the choice of $\pi^- \pi^-$, for which no strong resonance effects should mask correlations resulting from the production mechanism. It is therefore fortunate to see that the charge configuration which is most easily amenable for experimental study turns out to be of particular theoretical interes.

As reported previously,[12] for π^- production in pp collisions our approach reproduces correctly the values and energy dependence of $<n>$, $<n(n-1)>$ (and $f_2 = <n(n-1)> - <n>^2$) over the energy range 10-30 GeV/c. Further exploration of production mechanisms would follow from a measurement of the ratio $<n(n-1)>/<n>^2$ at high energies, since models differ widely on the energy dependence of this quantity.[22,23]

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[*] In defining such a ratio, we arbitrarily consider most heavy novas as due to single excitation. In this case there is neither a neat kinematical nor a clear dynamical separation between single and double excitation.

The success of our model in the 10-30 GeV/c range does not imply

solid predictions at NAL or ISR energies. High mass excitations

become more important at these very high energies, but the nova

production cross section and decay multiplicity are not determined

uniquely by the slow increase of <n> with energy, the only important

energy dependent input to our model. Predictions for the single-

particle distributions are safer, since their normalization is determined

by the already established slow increase of multiplicity.[25] The

normalization of $d^2\sigma/dy_1 dy_2$ increases in proportion to $\sqrt{s}$ in fragmentation

models. Predictions for NAL energies are given. These points are

discussed in Sec. III. Moments higher than <n(n-1)> would, of

course, also be interesting in the NAL ISR energy domain. Cosmic

ray results provide some hints of what to expect.[26, 27]

In Sec. IV we close with a few general remarks and predictions

about other easily accessible meson correlations. Calculations are

presented for K^+p (K^0, π^-) where one also has an exotic system.

II. Two-Particle Inclusive Distributions in the Nova Model.

In this section, we discuss our model calculation of two-particle

inclusive distributions at present machine momenta (10 to 30 GeV/c).

For definiteness, we examine the specific reactions $\pi^\pm p(\pi^-, \pi^-)$, but

most results are carried over easily to other processes. More details

are given also of previously reported results[12] on $pp(\pi^-, \pi^-)$. The two

final pions are labeled with indices 1 and 2. The two-particle inclusive

distribution is a function of 6 kinematic variables, which may be taken

to be the two longitudinal rapidities of the final pions, y_1 and y_2, in

the center-of-mass frame; two transverse momenta $|p_{T1}|$ and $|p_{T2}|$;

the angle ϕ between the two transverse vectors; and the center-of-mass

energy $\sqrt{s}$. In this Section we focus on y_1 and y_2 dependence, integrating

explicitly over p_{T1}^2 and p_{T2}^2. Thus, the quantity analyzed here is

$d^2\sigma/dy_1 dy_2$. Remarks are made in Section IV concerning dependence

on p_{T1}, p_{T2}, and ϕ. Energy dependence is discussed in Section III.

A. <u>Review of the Model</u>

As emphasized in the Introduction, the Nova model is

a phenomenological approach which embodies three features abstracted

from data on single-particle inclusive spectra.[3,4] We examine each

of these features in succession.

1. High energy collisions are termed "weakly inelastic"

in that one of the incident particles often retains a large fraction of its

center-of-mass incident energy. This fact is manifest in various ways.

For example, in pp(p) the final proton's longitudinal momen-

tum spectrum is observed to be flat,[19,28] or even to be slightly

peaked towards the kinematic limit $|p_L| = p_{L, max}$. In pion

production processes,[9,18] an excess of π^- in $\pi^- p(\pi^-)$ is observed

at $y \gtrsim 2$. This effect is seen in data displayed in Fig. 1, but shows

up more dramatically when data are plotted versus x, or versus y for

small p_T intervals.[18] The excess is associated with events in

which the π^- is quasi-elastically scattered, recoiling from an excited

proton-like system ("cluster" or "nova"). Small inelasticity already

implies some correlation among secondaries,[20] because secondaries

decaying from the excited cluster share a fraction of center-of-mass

energy which is substantially less than that which would be awarded

by energy-momentum conservation alone.

To represent small inelasticity in as obvious a fashion as possible,

we use single cluster excitation diagrams sketched in Fig. 2(a) and (b).

In addition, our fits[12, 22] to $d^2\sigma/dy_1 dy_2$ demonstrate the need for

contributions of the type shown in Fig. 2(c), in which both beam and

target are excited in a given event. Each nova is represented by an

excitation function $\rho(M)$, where M is the nova mass. An observed

pion in $\pi^+ p(\pi^-, \pi^-)$ or $pp(\pi^-, \pi^-)$ is a decay product of one of the nova

systems, but in $\pi^- p(\pi^-, \pi^-)$ it may also be a through-going quasi-

elastically scattered π^- (Fig. 2(b)).

 2. Single particle transverse momentum distributions

are observed to be damped sharply, following a gaussian like dependence

of the form $\exp(-\lambda p_T^2)$, where $\lambda \approx 3$ for large p_T^2, and $\lambda \approx 10$ for

small p_T^2 ($\lesssim 0.2$). Although often neglected, the distribution in p_T^2

has non-trivial implications for distributions in y. Indeed, at current

accelerator energies, the mean $<p_T>$ is not small compared to the

average energy available per secondary. Thus, the fact that a fair fraction of the available energy goes into transverse motion reflects itself in narrow distributions in x or y. We observe that over the energy range 10-30 GeV/c, $<p_T^2> \simeq <p_L^2>$ in the nova rest frame; this provides justification for our assumption that when clusters, produced with small inelasticity, decay by pion emission, the average decay distribution is approximately isotropic.[29] Another consequence is our assertion that the average number of decay pions n(M) is linearly proportional[30] to the nova mass M.

3. A third important feature of single pion inclusive spectra is the observed rapid approach to scaling[28] (energy independence) for intermediate values of $|x|$ ($|x| \approx 0.2$ to 0.6). These energetic secondary pions yet short of being leading particles, tend to result from decay of fairly low-mass novas. High-mass clusters, which can be excited with increasing energy, populate small values of $|x|$. Rapid scaling is thus achieved if our low M excitation spectrum $\rho(M)$ is energy independent and peaks at relatively small M. Energy independence at small M is supported by the observed[26] near constancy of σ_4 over the energy range 10 to 500 GeV/c; σ_4 is the four prong exclusive cross section.

These three crucial features of machine energy collisions are systematized in the Nova model, where either the beam or target particle is quasi-elastically excited into a superposition of resonance

states which subsequently decay by sequential emmision, with an isotropic decay distribution in the nova rest frame. The mean p_T of the pion distribution is related to the multiplicity of pions obtained from a nova mass M; this in turn fixes the normalization of $d\sigma/dy$. Although a few additional specific technical assumptions are made in setting up the Nova-model[21] calculation,* the general shapes of predicted distributions do not depend sensitively on these assumptions. The crucial assumptions are those stressed above. Insofar as energy dependence is concerned, the way in which we formulate the model leads to strong predictions for $<n(n-1)>$ in the NAL and ISR energy ranges. Energy variation is discussed in Section III, but seems immaterial for discussion of $d^2\sigma/dy_1 dy_2$ in the energy range 10 to 30 GeV/c. Although perhaps obvious, we stress that the model is not built upon what would be called fundamental theoretical postulates. Rather, we abstract simple observed features from single particle inclusive spectra and then imbed them in a framework which is convenient for further phenomenological analyses. Detailed calculations of shapes of single particle distributions are correct. Interesting tests are possible at the two particle inclusive level, where we can study whether additional dynamics is present, beyond that abstracted from the single particle spectra. Similarly our attitude toward single versus double excitation remains phenomenologically based[21] and no matter of principles.

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* As must be true in all approaches even weighted phase space.

B. Parametrization

      Relevant diagrams are sketched in Fig. 2. A nova is characterized by three quantities: its excitation spectrum $\rho(M)$, the mean number $n(M)$ of decay pions, and the average decay distribution $dD/d^3\vec{p}$ in the Nova rest frame. In addition, when considering leading pion effects, we must specify the momentum transfer (t) or transverse momentum (p_T) distribution of the leading particle. The t or p_T distribution of the whole cluster is irrelevant when we examine pions which are decay products of a cluster. Because of the small value of the pion mass and small Q value in the decay,[21] the convolution of p_T distributions for production and decay is controlled entirely by the decay distributions[31, 32] We discuss leading particle effects in Section II, E. Here we concentrate on parameterization of single nova excitation graphs Fig. 2(a) and (b).

      For reasons given above, the average pion decay distribution in the nova rest frame is chosen to have the symmetrical form

$$\frac{dD}{d^3p} = \frac{a}{k^2\omega} \exp\left(-\frac{p^2}{k^2}\right)$$

(2.1)

Here $\omega = (p^2 + \mu^2)^{1/2}$ is the energy of the secondary π in the nova rest frame; $p^2 = p_T^2 + p_L^2$; and μ is the pion mass. Parameter K is chosen to reproduce the correct average single particle p_T^2

distribution.[3,4] We determine a value K = 0.45 GeV/c, which

corresponds to a typical Q value of 330 MeV/c at each step in the

sequential decay of the nova. The distribution is normalized to unity

by a.

It will be noted that in Eq. (2.1) the same parameter K

controls both longitudinal and transverse momentum spectra, a result

supported by our comparisons with data. The form of the decay

distribution specifies a fixed average energy release per decay, so

that the total number n(M) of decay pions is linearly proportional to the

excitation mass[30] (M-M_o). The proportionality factor is also

related to the value of K, with reasonable values determined to lie

in the range 2.0 to 2.5 GeV^{-1}. After adjusting results to reproduce

the correct mean multiplicity[12] in pp(π^-) at 21 GeV/c, we find

$$n(M) = 2.1 \left(M - M_o \right) \tag{2.2}$$

This is understood as an implicit average over all decay chains; M_o

is the ground state mass of the nova. Although the factor 2.1 was

determined for proton novas, Eq. (2.2) is also used for our pion and

kaon nova calculations. Looking for key features, we minimize the

number of parameters.

In addition to Eq. (2.1), other forms for $dD/d^3\vec{p}$ are possible. We

also tried the expression [21]

$$\frac{dD}{d^3p} = \frac{a'}{K'^2\mu}\, exp\left(-\frac{p^2}{K'^2}\right)$$

$$(2.3)$$

This differs from Eq. (2.1) in that the factor ω^{-1} is absent on the right hand side. Here $K' = 0.33$ is different from K, for obvious reasons, but average quantities remain the same. Numerical calculations were performed with both Eqs. (2.1) and (2.3), and agreement was found, indicative of the fact that results in rapidity are not overly sensitive to the exact form for $dD/d^3\vec{p}$. All specific results reported here were done with Eq. (2.1). Neither Eq. (2.1) nor (2.3) provides a perfect representation of $d\sigma/dp_T^2$. To achieve better agreement, we should perhaps use a sum of gaussians on the right hand side[33] of Eqs. (2.1) and (2.3). Results in rapidity are insensitive to such precision; it is important but sufficient to reproduce the average behavior of $d\sigma/dp_T^2$.

Rewriting Eq. (2.1) in terms of rapidity, we obtain

$$\frac{dD}{dy_0\, dp_T^2} = \frac{\pi a}{K^2}\, exp\left(-\frac{p_T^2}{K^2}\right) exp\left(-\left(\frac{m_T\, sinh\, y_0}{K}\right)^2\right)$$

$$(2.4)$$

where y_0 is the rapidity in the nova rest frame and $m_T^2 = p_T^2 + \mu^2$. (Here we use the relations $p_L = m_T\, sinh\, y_0$ and $\omega = m_T\, cosh\, y_0$.)

For reasons given above, values of p_T are essentially identical in the center-of-mass and nova rest frames. To transform

rapidities, we use their convenient properties under Lorentz transformations. We write

$$y = y_0 \pm Y(s, M) \tag{2.5}$$

Here y is the center-of-mass rapidity. The (+) sign is taken for beamlike (e.g. meson) novas and the (-) sign for target novas. Nova rapidity Y is calculated from

$$\sinh Y = \frac{Q_N}{M} \tag{2.6}$$

where Q_N is the center-of-mass momentum of the nova.

$$Q_N = \left(s^2 - 2s(M^2 + M_p^2) + (M^2 - M_p^2)^2 \right)^{1/2} \Big/ 2\sqrt{s} \tag{2.7}$$

Here M_p is the mass of the "spectator" initial particle.

After integrating Eq. (2.4) over p_T , we obtain

$$\frac{dD}{dy_0} \approx \frac{a}{\cosh^2 y_0} \exp\left(-\left(\frac{M \sinh y_0}{k}\right)^2\right) \tag{2.8}$$

The form is approximate because in practice the integral must be cut off at finite p_T , in keeping with energy and momentum conservation. We define A(M, y) to be the normalized nova decay distribution in rapidity in the center-of-mass.

$$A(M,y) = \frac{dD}{dy_o}(y \mp Y) \qquad (2.9)$$

We turn next to the parameterization of our third quantity, the nova excitation spectrum $\rho(M)$. This quantity is the differential cross section $d\sigma/dM$, integrated over momentum transfer, for production of a nova of mass M. Choice of a functional form for $\rho(M)$ is necessarily somewhat ad hoc.[21] However, because of the averaging (integration) over M required in calculating various distributions, the distribution of secondary pions coming from nova decay is insensitive to details of $\rho(M)$. Our form for $\rho(M)$ rises from threshold $(M=M_o)$, peaks at ≈ 1 GeV above threshold, and falls in proportion to $(M-M_o)^{-2}$ for large M. As long as this general rise and fall of $\rho(M)$ is maintained, distributions are not modified greatly.

The asymptotic M^{-2} behavior is suggested by Regge theory.[34] Given our determination that $n(M) \propto M$, the M^{-2} behavior is also required in order that the mean pion multiplicity grow in proportion to log s. This logarithmic property is already enough to guarantee a reasonable phenomenological description of single-particle spectra near $x = 0$.[25] However, two-particle distributions are sensitive to high mass excitations; therefore, our estimates of high M behavior are not guaranteed to account correctly for normalization

of $d^2\sigma/dy_1 dy_2$.

The form for $\rho(M)$ is different in general for baryon and meson Novas. The specific form we choose is[21]

$$\rho_\kappa(M) = (M-M_\kappa)^{-2} \exp\left(-\beta\kappa/(M-M_\kappa)\right)$$ (2.10)

The values β_p = 2 GeV for proton excitations and β_π = 2.4 GeV/c for pion excitations are found to give good agreement with both single- and two-particle inclusive spectra in $pp(\pi^-, \pi^-)$ and $\pi^\pm p(\pi^-, \pi^-)$. We normalize our calculation to the observed total inelastic cross section at a given energy. This introduces normalization constants C_α. In addition, to reproduce the fact that secondaries tend to follow the incident meson (an observed asymmetry of single-particle distributions), we set C_π/C_p = 2, as in Ref. 21 and 22.

If only single nova excitations contribute to $\pi p(\pi)$ (e.g. Fig. 2(a) and (b) only), then the inelastic cross section is given by

$$\sigma_{inel}^{(1)} = C_0 \left(\int_{m_p}^{\sqrt{s}-m_\pi} \rho_p(M)dM + 2 \int_{m_\pi}^{\sqrt{s}-m_p} \rho_\pi(M)dM \right)$$ (2.11)

and

$$\sigma_{inel}^{(1)} \langle n \rangle^{(1)} = C_0 \left(\int_{m_p}^{\sqrt{s}-m_\pi} n_p(M)\rho_p(M)dM + 2 \int_{m_\pi}^{\sqrt{s}-m_p} n_\pi(M)\rho_\pi(M)dM \right)$$ (2.12)

Note that $\langle n \rangle$, the average number of pions, is independent of C_0. As already stressed,[21] our keeping the same expression all the way to phase space limits in purely phenomenological. It enforces the correct multiplicity but should not be given a dynamical significance. Our crude spectrum ignores low energy resonances (ρ^+, Δ^+). They do not contribute to the π yield but we lump into single excitation cases in which the $\pi(p)$ recoils as a Δ) while exciting the other particle.

The quantity n(M) gives the average number of pions which decay from a nova of mass M. These pions must be apportioned among the three possible charge states, a procedure detailed in Section III. Here we denote by $n_\alpha^-(M)$ the average number of negative pions expected from the decay of an α nova of mass M. We introduce $<n_\alpha^-(n_\alpha^- - 1)>_M$ which is twice the average number of pairs of π^- expected from decay of an α nova of mass M.

The contents of this subsection are summarized in the following expressions for the single nova contributions to $d\sigma/dy$ and $d^2\sigma/dy_1 dy_2$ for π^- production in $M_1 M_2 (\pi^-, \pi^-)$.

$$\frac{d\sigma^{(1)}}{dy} = \sum_{\alpha=1}^{2} C_\alpha \int_{M_\alpha}^{\sqrt{s}-M_\beta} \rho_\alpha(M) \, n_\alpha^-(M) \, A(M, y) \, dM \tag{2.13}$$

$$\frac{d^2\sigma^{(1)}}{dy_1 dy_2} = \sum_{\alpha=1}^{2} C_\alpha \int_{M_\alpha}^{\sqrt{s}-M_\beta} \rho_\alpha(M) \, <n_\alpha^-(n_\alpha^- - 1)>_M \, A(M, y_1) \, A(M, y_2) \, dM \tag{2.14}$$

These equations were used to obtain distributions presented in Ref. 22 for $\pi^+ p(\pi^-, \pi^-)$. As remarked in that paper, the fact that both π^- are always associated with the same nova results in a marked tendency for $d^2\sigma/dy_1 dy_2$ to peak at positive values of y_2 when y_1 is positive. To the extent that this strong feature is not observed directly in data for large values of $|y_1|$ it is obvious that, the two observed π^- sometimes originate from two different clusters. In a configuration such

as that shown in Fig. 2(c), the two novas may decay with pions being

emitted in a back-to-back fashion. In the next subsection, we discuss

our parameterizations of Fig. 2 (c).

C. Double nova Production.

If we insist that one π^- is produced with large $|y|$, kinematics

suggest that it originates from a light fast nova. Because the probability

is small for a nova with small M to give two π^-, it is likely that if a

second π^- is observed, it will have come from a different cluster,

moving in the opposite direction from the first. Two particle distri-

butions $d^2\sigma/dy_1 dy_2$ with one y large are sensitive to this type of

double nova excitation (as previously remarked.[22]) It remains

however, that single nova excitation (one nova per event) is important.[35]

The width (or dispersion) of $d\sigma/dy$ is determined by low

mass novas, which can seldom produce two pions of the same charge.

Heavier novas, which can give two π^-, are restricted by kinematics to

relatively small values of y. Thus, decay pions from heavy novas

also occupy the small y region. To the extent that single nova effects

dominate , $d^2\sigma/dy_1 dy_2$ will be more narrow than $d\sigma/dy$. This is

supported by the data in Fig. 3, for small values of y_1. However,

viewed as a function of y_2, $d^2\sigma/dy_1 dy_2$ at large y_1 is as broad as

$d\sigma/dy$, as expected if two cluster contributions are now significant.

Our physical picture then is one in which dominant single nova effects

supply the bulk of the distributions, particularly at small y. Their

contribution is supplemented by double-nova effects, whose

role is particularly important in $d^2\sigma/dy_1 dy_2$ when either or both y

are large.

When two clusters are formed in a given event, it is not

obvious that we should describe their excitation and decay in the same

way as we handle the single nova case. However, to the extent that the

masses of the clusters are small, their pion yields should be similar

to those of novas formed in a single excitation configuration. We

therefore adopt the same $\rho(M)$, $n(M)$, and so forth.

For the double excitation contribution to σ_{inel}, we write

$$\sigma_{inel}^{(2)} = \int \rho_1(M_1) \rho_2(M_2) R(M_1, M_2) dM_1 dM_2 \tag{2.15}$$

Here M_1 and M_2 are masses of the two novas. Function $R(M_1, M_2)$

scales the double excitation cross section as compared to the single

excitation contribution, Eq. (2.11). We find it adequate to take a

constant value for R, followed by a sharp cutoff (R = 0) if momentum

transfers $t(M_1, M_2; s)$ are larger than 1 $(GeV/c)^2$.

For the double excitation contribution to <n>, we derive

$$\sigma_{inel}^{(2)} \langle n \rangle^{(2)} = \int \rho_1(M_1) \rho_2(M_2) R(M_1, M_2) \left(n_1(M_1) + n_2(M_2) \right) dM_1 dM_2 \tag{2.16}$$

For $\pi^+p(\pi^-)$, $pp(\pi^-)$, $K^+p(\pi^-)$, therefore,

$$\langle n^- \rangle = \frac{\langle n \rangle^{(1)} \sigma_{inel}^{(1)} + \langle n \rangle^{(2)} \sigma_{inel}^{(2)}}{\sigma_{inel}} \qquad (2.17)$$

where $\sigma_{inel} = \sigma_{inel}^{(1)} + \sigma_{inel}^{(2)}$. The corresponding double excitation

contribution to $d\sigma/dy$ is

$$\frac{d\sigma^{(2)}}{dy} = \int \rho_1(M_1)\, \rho_2(M_2)\, R(M_1,M_2)\, dM_1\, dM_2 \quad \times \qquad (2.18)$$
$$\times \Big(n_1(M_1)\, A_1(M_1,y) + n_2(M_2)\, A_2(M_2,y) \Big)$$

The decay functions $A_i(M, y)$ are the same as those discussed previously

(Eq. (2.9)), except that in the kinematics the spectator mass is now that

of the bachelor nova. The two terms appearing in Eq. (2.18) represent

the fact that the observed π may originate from either of the two clusters

formed in the event.

For $d^2\sigma/dy_1 dy_2$, we have three terms, which refer to our

associating the two observed π to two different clusters or to the same

one. Specifically,

$$\frac{d^2\sigma}{dy_1\, dy_2} = \int \rho_1(M_1)\, \rho_2(M_2)\, R(M_1,M_2)\, dM_1\, dM_2 \quad \times$$
$$\times \Big\{ n_1(M_1)\, n_2(M_2) \Big(A_1(M_1,y_1)\, A_2(M_2,y_2) + A_1(M_1,y_2)\, A_2(M_2,y_1) \Big)$$
$$+ \langle n_1(n_1-1) \rangle_{M_1}\, A_1(M_1,y_1)\, A_1(M_1,y_2)$$
$$+ \langle n_2(n_2-1) \rangle_{M_2}\, A_2(M_2,y_1)\, A_2(M_2,y_2) \Big\} \qquad (2.19)$$

Furthermore,

$$\sigma_{inel}^{(2)} \langle h(n-1) \rangle^{(2)} = \int \rho_1(M_1) \rho_2(M_2) R(M_1, M_2) dM_1 dM_2 \times$$

$$\times \left\{ 2 n_1(M_1) n_2(M_2) + \langle n_1(n_1-1) \rangle_{M_1} + \langle n_2(n_2-1) \rangle_{M_2} \right\}$$

$$(2.20)$$

D. Normalization and Calculated Distributions.

The necessary ratio of $\sigma^{(2)}/\sigma^{(1)}$ [c.f. Eqs. (2.11) and

(2.15)] is determined by fitting shapes of distributions for $\pi^+ p(\pi^-, \pi^-)$

shown in Fig. 3. The essentially perfect agreement with data is

obtained with the value $\sigma^{(2)}/\sigma^{(1)} = 3/7$. As mentioned above, the

double excitation "correction" to dominant single excitation is needed

in order to reproduce the width of $d^2\sigma/dy_1 dy_2$ when one rapidity is

kept large.

The single particle distribution $d\sigma/dy$ for $\pi^+ p$ (π^-) at 18.5

GeV/c is shown in Fig. 1. Again, agreement is excellent. Given our

determination of $\sigma^{(2)}/\sigma^{(1)}$, we normalize our calculation to the

observed inelastic cross section of 21 mb. Thus, normalizations of

$d\sigma/dy$ in Fig. 1 and $d^2\sigma/dy_1 dy_2$ in Fig. 3 are predictions of the model.

A similar procedure was pursued[12] for $pp(\pi^-, \pi^-)$, a

cleaner process to analyze inasmuch as only proton novas contribute and the single particle distribution $d\sigma/dy$ is symmetric about $y = 0$. Again, a 7/3 ratio of single to double excitation was found to provide excellent agreement with the Michigan State pp data. Results from Ref. 12 are reproduced here as Fig. 4. Also shown in Fig. 4 is the explicit contribution from double excitation to $d^2\sigma/dy_1 dy_2$ versus y_2, for $y_1 \simeq 2.0$. The pp calculation is normalized to the observed pp inelastic cross section of 30 mb at 21 GeV/c; normalization of $d\sigma/dy$ and $d^2\sigma/dy_1 dy_2$ are predicted correctly.[12]

Two particle distributions are more narrow than single distributions, and thus give obvious indication of strong positive clustering (correlation) of secondary π^-'s. However, this effect simply reflects the fact that $d^2\sigma/dy_1 dy_2$ receives contributions from heavier nova events yielding at least two π^- (6 prongs or more in pp or π^+p), whereas $d\sigma/dy$ is populated by a larger sample of events. Over the energy range 10 to 30 GeV/c, the four prong cross section (only one π^-) is particularly large. In terms of the model, production of two π^- requires relatively heavy clusters, thus narrow distributions in y of the decay pions. From a purely experimental point of view, the same conclusion may be reached by comparing the width of $d^2\sigma/dy_1 dy_2$ to that of $d\sigma/dy$ limited to events in which at least two π^- are found. Widths are then comparable.[12]

Another interesting way to examine clustering is to define

an associated multiplicity[12, 36]

$$\langle n(y)-1 \rangle = \frac{\sum_n n(n-1) \frac{d\sigma_n}{dy}}{\frac{d\sigma}{dy}} = \frac{\int dy' \left(d^2\sigma / dy' dy \right)}{\frac{d\sigma}{dy}} \qquad (2.21)$$

Because the averaged double distribution is narrower in y than the

single particle distribution, Eq. (2.21) should have a maximum near

y = 0. Results of our calculations are compared with Notre Dame

data in Fig. 5. Agreement is excellent. This again indicates,

through a slightly different test, that our description of the clustering

effect is enough to explain observed positive correlations among

π^-'s. The observed effect, particularly marked between distributions

requiring only one versus those requiring at least two π^-, is due to

the relative importance of low mass novas. This effect disappears

slowly with increasing n, when one compares distributions limited to

multiplicity with more than n and n + 1 π^-'s respectively.

E. Leading Particle Contribution.

If one of the observed secondaries can be a quasi-elastically

scattered particle, as in $\pi^- p(\pi^-)$, an additional corresponding term

should be added to the single and double distributions. The leading

particle rapidity distribution is obtained in terms of the excitation

spectrum of the recoil nova; it reads

$$\frac{d\sigma_L}{dy} = k \sqrt{s} \int \frac{1}{M_0} \frac{d\rho(M_0)}{dt} dp_T^2 \qquad (2.22)$$

The center of mass momentum is denoted by k; and M_0, the recoil nova mass, is readily evaluated in terms of y and p_T^2, as

$$(2.23)$$

$$M_0^2 = s - 2\sqrt{s}\, m_T \sinh y + \mu^2$$

$$\text{with} \qquad m_T^2 = p_T^2 + \mu^2$$

For phenomenological reasons we have taken

$$\frac{d\rho}{dp_T^2} = \rho(M)\, B \exp\left(-B\, p_T^2\right) \qquad (2.24)$$

with $B = 4 \text{ GeV}^{-2}$ independent of M.

At fixed y, different values of p_T^2 correspond to different values of M. Since y depends very strongly on p_T^2 at fixed M, no prominent quasi-elastic peak is obtained (Fig. 1). In order to see the peak clearly, one has to consider separately small p_T^2 intervals.[18]

Keeping only single nova excitation and normalizing the calculated distribution to the total inelastic cross section we obtain too strong a leading particle contribution.[21] This is another indication that some double excitation is also present. Granting only 70% of σ_{inel} to single excitation (as for $\pi^+ p$) we decrease the

calculated value of the leading pion distribution and obtain a good agreement with the data shown on Fig. 1. Our estimate of the leading particle effect could be an overestimate.[17] The ratio $\sigma^{(2)}/\sigma^{(1)}$ which we used is a compromise which could be modified were the shape and normalization of the leading particle contribution to change with more accurate data.[37] As already emphasized[21] it would be very interesting to examine data on the slow proton recoil distribution which should offer the most reliable determination of the actual strength of single excitation.

To obtain $d\sigma/dy$ for $\pi^-p(\pi^-)$ we add three contributions, from Eqs. (2.13), (2.18), and (2.22). Our calculation for $\pi^-p(\pi^-)$, shown in Fig. 1, is normalized to $\sigma_{inel}(\pi^+p)$! The relative normalization for $\pi^\pm p$ is a prediction of the model. The discrepancy with data seen for $y < o$ (for π^-p) is related to charge exchange effects neglected in our calculation.[21] Such effect which should fall with increasing energy (as opposed to the large difference found at low positive y) are much larger for π^-p than for π^+p processes.

The leading particle also contributes to double distributions. Indeed for $y_1 > 2$, the observed π^- is in general the quasi-elastically scattered π^-. One finds

$$\frac{d^2\sigma}{dy_1 dy_2} = \int \frac{k\sqrt{s}}{M_0} \frac{d\rho(M_0)}{dt} n_p(M_0) A_p(y_2, M_0) dp_T^2 \qquad (2.25)$$
$$+ \left(1 \leftrightarrow 2\right)$$

where A_p (y_2, M_o) is the pion distribution obtained from the proton

nova and $n_p(M_o)$ is the average number of π^- from a proton nova of

mass M_o. Depending on the magnitude and sign of y_1, only one term

in (2.25) may be significant in the double distribution.

Upon including contributions from Eqs. (2.14), (2.19), and

(2.25), we obtain results presented in Fig. 6. Although overall

agreement with data is less spectacular than for $\pi^+ p$ and pp, results

are acceptable for $y > -0.8$. For $y < -0.8$, normalization difficulties

are particularly evident. However, these can be traced to the prob-

lem which we experienced for $y < 0$ in $d\sigma/dy$, and which we discussed

above. Improving parameters was not attempted.

Our results for $\pi^\pm p$ and pp processes show that the simple

features we abstracted from single particle inclusive data and imbedded

in the nova framework are fully sufficient for reproducing more detailed

information provided by two particle inclusive reactions. The clustering

effect (small inelasticity) inferred from single particle spectra plays

an especially significant role at the two particle inclusive level.

This is an important result·, but, at the same time, we may conclude

that beyond confirming features of single particle data, present data

on $d^2\sigma/dy_1 dy_2$ do not reveal any new specific dynamic phenomenon.

F. Correlation Function.

It may seem practical to summarize the information which is contained in the double distributions in terms of a correlation function $C(y_1, y_2)$. However, its interpretation may not be too transparent at energies where the mean multiplicity of the observed secondaries is 1 or 2! Consequently the shape and normalization of the correlation are quite sensitive to its definition. The conventional definition is

$$(2.26)$$

$$C(y_1, y_2) = \frac{1}{\sigma_{inel}} \frac{d^2\sigma}{dy_1 dy_2} - \frac{1}{\sigma_{inel}^2} \frac{d\sigma}{dy_1} \frac{d\sigma}{dy_2}$$

At present machine energy however, the second term measured in $pp(\pi^-)$ collision for instance, has a strong contribution from 4 prong events which make no contribution to the first term. As a result one merely sees the difference between two large and weakly correlated terms. This can hardly be called a correlation in a dynamical sense. Calculating $C(y_1, y_2)$ from the double and single distributions is a very inefficient way to display the available information.

At the same time (2.26) is too sensitive to the type of reaction or particle species. The first term normalizes to $<n(n-1)>$ when the second one normalizes to $<n>^2$, and it is merely the charge dependence of these normalizations which leads to such different looking correlation functions.[9-11] Indeed from Table I we see that $C(y_1, y_2)$ for $\pi^- p (\pi^-, \pi^-)$ should be about three times that of

$\pi^+ p (\pi^-, \pi^-)$ even though the basic production mechanisms are the same in both cases. It seems appropriate to at least partly eliminate such effects, despite all the evils attached to still a new definition for the correlation function. We therefore define

$$C(y_1, y_2) = \frac{1}{\sigma_{inel} \langle n(n-1) \rangle} \frac{d^2\sigma}{dy_1 dy_2} - \frac{1}{\sigma_{inel}^2 \langle n \rangle^2} \frac{d\sigma}{dy_1} \frac{d\sigma}{dy_2} \tag{2.27}$$

We have computed this correlation function and compared it with the data.[9] As expected from our agreement with data in Figs. 1, 3, and 5 we reproduce very well the observed a priori puzzling curves . The correlation curves are shown on Figure 7. We have used (2.27), and $C(y_1, y_2)$ integrates to zero. This would also be the case were we to use (2.26) with a Poisson distribution as a good approximation.

III. Pion Multiplicity and Normalization of Correlations.

A. Charge Ratios.

Novas are postulated to decay by sequential emission of pions, a property in keeping with their resonance like character. The average number of decay pions is determined by the nova mass, Eq. (2.2). Quantity $n^-(M)$ in Eqs. (2.13) and (2.18) is the average number of π^- coming from all decay chains of a nova of mass M; $\langle n^-(n^- - 1) \rangle_M$ in Eq. (2.14), is twice the average number of pairs of

π^-. For very large multiplicity, a statistical limit may be approached in which $(\pi^+\pi^-)$ pairs are twice as frequent as $(\pi^0\pi^0)$ pairs. Extimates of the relative number of π^+, π^-, and π^0 at low multiplicity depend crucially on the total charge of the nova. Values can be bounded by isospin arguments, but are nonetheless somewhat arbitrary. Results given below are representative of typical decay chains. The appropriateness of our procedure is verified by the fact that we predict approximately the correct normalization of $d\sigma/dy$ for $\pi^- p(\pi^-)$ at positive y in terms of that for $\pi^+ p(\pi^-)$ (c.f. Fig. 1).

For large multiplicity, we consider a binomial distribution with $2/3$ probability for a $(\pi^+\pi^-)$ pair and $1/3$ probability for a $\pi^0\pi^0$) pair. A straightforward calculation gives

$$n^-(M) = \frac{n(M)}{3} \equiv \frac{N}{3} \tag{3.1}$$

for the average number of π^-. Here $N \equiv n(M)$ is the average number of pions of all charges (taken as $n(M) = 2.1 \, (M - M_0)$ in Section II). The average number of pairs of π^- is

$$\langle n^-(n^- - 1) \rangle_M = \frac{N(N-2)}{9} \tag{3.2}$$

For low multiplicity (small M), charge effects are relevant. Proton and π^+ novas produce fewer π^- than Eq. (3.1) and π^- novas give correspondingly more. For positively charged novas, reasonable interpolations are

$$n^-(M) = \frac{N-1}{3}$$

(3.3)

and

$$\langle n^-(n^--1) \rangle_M = \frac{(N-1)(N-3)}{9}$$

(3.4)

Comparisons with some explicit decay chains show that these approximations are quite decent. In calculations of $\pi^+ p\,(\pi^-, \pi^-)$ and $pp(\pi^-, \pi^-)$ reported in Section II, Eqs. (3.3) and (3.4) were employed.

For negatively charged novas, one extra π^- should be added to the right hand side of Eq. (3.3), a result obtained most simply from a consideration of charge conservation. Thus, for the number of π^- from π^- novas, we use

$$n^-(M) = \frac{N+2}{3}$$

(3.5)

and

$$\langle n^-(n^--1) \rangle_M = \frac{(N+3)(N-1)}{9}$$

(3.6)

Our estimates are crude, but from an examination of explicit decay chains, we estimate a 20% reliability in $\langle n(n-1) \rangle_M$ for $N \leq 6$, adequate for our purposes. Inasmuch as all quantities of interest are computed averages over $n(M)\,\rho(M)$ or $\langle n(n-1) \rangle_M \rho(M)$, a more refined approach is not justified until $\rho(M)$ is given a more

exact form.

As described in Section II, Eqs. (3.3) to (3.6) lead to good

agreement with shapes and normalization of $d\sigma/dy$ and $d^2\sigma/dy_1 dy_2$

at fixed s. We now turn to the energy dependence of $<n>$ and

$<n(n-1)>$, quantities which normalize $d\sigma/dy$ and $d^2\sigma/dy_1 dy_2$ (c.f.

Eqs. (1.1) and (1.2), respectively. **The** quantity $<n(n-1)>$ is **more**

model dependent than the shapes of spectra.

B.   Energy Dependence.

Our calculated values of $<n>$ and $<n(n-1)>$ are compared

with 18.5 GeV/c $\pi^{\pm}p$ data in Table I. Calculated values of $<n>$ and

$f_2 = [<n(n-1)> - <n>^2]$ are compared with pp data in Figs. 8(a) and (b).

Agreement with $\pi^{\pm}p$ and pp data in the 10-30 GeV/c energy range is

quite respectable.

As remarked in Section II, our model is constructed to give

a mean multiplicity $<n^->$ which grows logarithmically. Indeed, at

asymptotic energies, we predict $<n^-> \sim 0.7 \log s$. For $<n(n-1)>$, the

nova model, in common with all the diffractive fragmentation models,[2,29]

predicts an asymptotic increase with energy of the form $<n(n-1)> \propto \sqrt{s}$.[*]

By contrast, in multiperipheral[38,39] and other weak correlation

models $<n(n-1)> \propto \log s$, or $\log^2 s$.[8]

Referring to our discussion of the nova model in Section II.A, we

observed that the prediction $<n(n-1)> \propto \sqrt{s}$ is actually not demanded

by our phenomenological input. Beginning with the fact that symmetry

_____

[*] In our model $<n(n-1)>_M \rho(M)$ goes to a constant at large M; thus
$<n(n-1)> \propto \int dM = \sqrt{s}$.

in the nova decay frame suggests $n(M) \propto M$, we chose $\rho(M) \propto M^{-2}$ to

guarantee $<n> \propto \log s$ (and an asympotically energy independent σ_{inel}).

However, symmetry is almost forced on us by small inelasticity and

the universal p_T dependence only for reasonably small p_{lab} (<100 GeV/c),

i.e. for M not infinitely large. We have no way of knowing whether

$n(M) \propto \log M$ (and $\rho(M) \sim M^{-1}(\log M)^{-1}$) would be more appropriate

at very large M.[40] This form would be suggested if the "decay"

takes on a one-dimensional character[20] in the nova rest frame, as

might be appropriate at very large s where $<p_L> >> <p_T>$.*

For reasons of phenomenological simplicity, we stick to

$\rho(M) \propto M^{-2}$, for all M at all values of s. Thus, we are able to make

predictions not only for asymptotic s dependence of $<n>$ and $<n(n-1)>$,

but we can also give specific values for these quantities in the ISR and

NAL energy ranges. Some support for the relevance of fragmentation

models at ISR energies comes from the observation that in pp collisions

the large (π^+/π^-) ratio is consistent with being energy independent at

$0.25 \lesssim x \lesssim 0.3$ between 24 and 500 GeV/c.[28, 41] This observation

has, however, very little implication for multiplicity growth.

Predicted values for $<n>$ and $<n(n-1)>$ for $\pi^{\pm}p$ at 100 GeV/c

are given in Table II. Here n always refers to π^- production. For pp

collisions, results given as a function of energy in Figs. 8(a) and (b),

are contrasted with available data.[42] Note that our predicted multiplicity

distribution, which is narrower than Poisson ($f_2 < 0$) in the energy

*If this is true f_2 would increase logarithmically after an inital sharper
 rise.

range $p_{lab} <$ 30 GeV/c, has become broader than Poisson ($f_2 > 0$)

by 100 GeV/c. This occurs for both pp and $\pi^{\pm}p$.

In Fig. 8, our values for $<n>$ and $<n(n-1)>$ are in reasonable

agreement with preliminary Serpukhov data[43] at 69 GeV/c. For

$<n>$, we are one to two standard deviations too high in comparison

with Echo Lake data[26,44] and too low in comparison with ISR data.[45]

However, both experiments are subject to some systematic error.

Our most obvious disagreement with present data is seen clearly in

Fig. 8(b), where we are a factor of 5 or 10 standard deviations too

high in comparison with Echo Lake results[44] on f_2 at 200 GeV/c. If

these Echo Lake data are correct, they would seem to rule out

absolutely the predicted s dependence $<n(n-1)> \propto \sqrt{s}$. As remarked

above, this would eliminate all pure fragmentation models,[2,29] unless

they abandon claim on present machine energy results. Our quantitative

predictions for $<n(n-1)>$, based on specific fragmentation model

which reproduces data in the 10-30 GeV/c range very well, underscore

the crucial importance of early measurements of $<n(n-1)>$ at NAL

and ISR.[22,23] More important than statistical accuracy is the obvious

need to avoid systematic bias for cross-sections with large prong

number, a bias which could significantly raise or lower $<n(n-1)>$.

IV. Other Correlations and Discussion.

The general ideas and specific procedures described in Sec. II

and III can be applied easily to other cases to check if the picture of particle production which has been incorporated into the Nova model is generally applicable. We therefore comment on a number of different correlation experiments.

Our detailed study of two π^- correlations was motivated by the availability of experimental information and supported by the special interest offered by an exotic system. There are no strong final state interactions superimposed on the production process proper. This latter point would also apply to $K^+p(K^o, \pi^-)$ reactions. The relatively low yield of K^o from proton novas, compared to K^+ novas, causes the distributions to be shifted to positive kaon rapidities but, otherwise, many features, already found in two π^- correlations, should also be met. We have calculated double rapidity distributions for the reaction $K^+p(K^o, \pi^-)$ at 16 GeV/c and present them in Fig. 9. We have used the equations of Section II and III. The number of K^o obtained from K^+ and proton novas respectively are taken to be[46]-

$$n_{K^+}^{K^o} (M) = \frac{1}{2}$$

(4.1)

$$n_p^{K^o} (M) = 0.06 + 0.014 \, (M - M_{th})$$

Where M_{th} is the threshold mass for decay of a proton into a K^o and

Σ^+ system. The Kaon excitation spectrum is assumed to have the same general shape already used for pion and proton excitations. Just as for pion induced reactions, the ratio C_K/C_p is 2. We present this calculation in order to stimulate a more detailed comparison with possible data. Hypercharge exchange should provide relatively larger correction terms than those met with pion induced reactions (Fig. 1).

Correlations for $\pi^+\pi^-$ or $\bar{K}^0\pi^-$ systems would be interesting. They should show some resonance effects but confirm the main features already found for two pions with the same charge. At present machine energy $\pi^+\pi^-$ double distributions will be normalized at a much larger value. Correlations between final baryon and meson (or two baryons in pp collisions) would also be extremely useful. They would in particular provide further insight on the leading particle effect.

The use of rapidities instead of the Feynman scaling variable x has some obvious kinematical advantages since the peak at small rapidities is compressed into an even sharper peak in an (x_1, x_2) distribution. On the other hand, the x variable is better for examining leading particle effects because of the more accurate determination of the nova mass for fixed x, expected after averaging over p_T^2. In any case the strongly peaked shape of the two pion distributions in

x_1 and x_2 may show directly some interesting features.[47] Because

of our success with the rapidity distribution we expect an equally

good quantitative representation of the main features when displayed

in terms of x_1 and x_2. In particular we may draw contour maps

in x_1, x_2 for a 3 dimensional presentation of the logarithm of the

inclusive double distribution. It is remarkable that at 16-20 GeV we

obtain an approximately pyramidal structure at relatively low x

values with roughly planar surfaces in the separate quadrants

$(x_1 \gtrless 0; x_2 \gtrless 0)$.

Our model does not predict dynamical correlations between the

transverse momenta of the secondaries (two π^- in section II). The

nova decay is statistical and the inclusive distributions approximately

factorize in y and p_T^2. As a result $d^2\sigma / dp_{T1}^2 \, dp_{T2}^2$ should generally

peak at $p_{T2}^2 = 0$ for any small value of $p_{T_1}^2$. Nevertheless, if we

associate secondaries with large p_T^2 to long jump decays,[33] some

correlations could show up. Since long jump decays are relatively

infrequent down each decay chain, the $p_{T_2}^2$ distribution at fixed and

relatively large $p_{T_1}^2$ should be narrower then when averaged over all

observed $p_{T_1}^2$ values. In general however little correlation is expected

besides that imposed by energy conservation.

Correlations in ϕ have been studied in several cases already

$K^+ p(K^0, \pi^-)$[10] $K^- p(\overline{K}^0, \pi^-)$[48] and $pp(\pi^-, \pi^-)$.[12,49] In all reactions

little correlation in ϕ has been observed beyond that expected from phase space considerations. Reported results are in agreement with what expected from the Nova model.[49] This may test our sequential decay picture but the observed features are presumably even more general.

As p_T^2 is increased, the distribution should fall more rapidly in y, at least in the range of validity of Eq. 1 ($p_T^2 < 0.3$). Thus, the rapidity correlations restricted to larger p_T^2 ($p_T^2 > 0.1$) should be much stronger (fall off more sharply with y) than those for smaller p_T^2. This expectation is almost obvious from the single particle rapidity distribution as a function of p_T^2. This effect is observed experimentally.[9]

In conclusion, it appears that the small inelasticity, rapid scaling and universal p_T^2 distributions observed at present accelerator energies are adequate for understanding the key features of correlation data. No further dynamical feature seems to be of any relevance within present accuracy. The Nova model, which embodies this features and provides a practical calculation scheme, appears successful. As already stressed, measurements of <n>, and in particular of <n(n-1)>, performed with increasing energy would provide extremely useful information. The strong clustering effect, ascertained at present machine energy, may continue to hold at NAL energy or gradually disappear into a multi-clustering mechanism with weaker correlations which could become dominant at much higher energies.

ACKNOWLEDGEMENT

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[23] This ratio differentiates strongly correlated fragmentation models where it increases with energy as $\sqrt{s}/\log^2 s$ from weakly correlated (multi-peripheral) models where it tends to a constant with increasing energy. This point has been emphasized by G. F. Chew and C. N. Yang.

[24] We use throughout the short hand notation AB(C), for $A+B \rightarrow C+$ Anything,

and AB(C, D) for A+B→C+D+Anything.

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[42] In all calculations reported in this section, we retain a 7/3 ratio for single to double excitation. Except for leading particle effects, distributions are not extremely sensitive to its value which could be varied slightly.

[43] Soviet-French Collaboration, preprint (1972) - Mirabelle data.

[44] To obtain $<n^->$ from the results of Ref. 26 and 43, we take $<n^-> = 0.5(<n^{ch}> - 2)$. Values of f_2 are computed from tables of prong cross sections given in these two papers.

[45] G. Neuhofer et al., Phys. Letters $\underline{37}$B, 438 (1971) and Phys. Letters $\underline{38}$B, 51 (1972). From this $pp(\gamma)$ experiment, $<n^o>$ is obtained. We then assume that $<n^-> = <n^o>$, which should not be unreasonable for large values of the multiplicity, although $<n^-> = <n^o> - 0.3$ might be better. We are grateful to G. Charlton (Argonne) for his comments on the γ data.

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FIGURE CAPTIONS

Figure 1   Single particle inclusive rapidity distribution for

$\pi^{\pm}p(\pi^-)$ at 18.5 GeV/c. Data are from Notre Dame (Ref. 9).

Solid curves are Nova model calculations described in the text.

Calculations are normalized to the π^+p inelastic cross section:

21 mb. The relative normalization of the two distributions is

a prediction of the model. The difference between the two sets

of data at large negative rapidity (proton fragments) is due to

charge exchange effects not included in our model calculation.

We also did not include some rho production obviously effective

at large rapidity.

Figure 2   Three contributions to the inelastic cross section. In (a)

an incident meson is excited into a meson nova. The incident baryon

loses momentum but is not excited. In (b), the incident baryon is

excited into a baryon nova but the meson is quasi-elastically

scattered. In (c), both meson and baryon are excited. As a first

approximation we keep only terms involving no quantum number

exchange.

Figure 3   Two π^- inclusive distributions observed in

$\pi^+p(\pi^-, \pi^-)$ at 18.5 GeV/c. Data are from Notre Dame (Ref. 9).

$d^2\sigma/dy_1 dy_2$ is plotted versus y_2 for the following intervals of y_1:

On the left

a) $-0.2 < y_1 < 0.2$; b) $-0.6 < y_1 < -0.2$; c) $-1.0 < y_1 < -0.6$; d) $-1.4 < y_1 < -1.0$;

e) $-1.8 < y_1 < -1.4$; f) $-2.2 < y_1 < -1.8$ and g) $-2.6 < y_1 < 2.2$. On the right

h) $0.2 < y_1 < 0.6$; i) $0.6 < y_1 < 1$; j) $1 < y_1 < 1.4$; k) $1.4 < y < 1.8$;

l) $1.8 < y_1 < 2.2$; m) $2.2 < y_1 < 2.6$ and n) $2.6 < y_1 < 3$.

Solid curves are obtained from the Nova model with 70% of

σ_{inel} associated with single nova excitation $\left[\text{Fig. 2(a) and (b)}\right]$; the

rest is double excitation (Fig. 2(c)). Only some of the curves have

been drawn.

Figure 4   Experimental distributions and Nova model calculations

for $pp(\pi^-, \pi^-)$ at 21 GeV/c. Data are from Ref. 12. Double

distributions are plotted versus y_2 for values of y_1 in the intervals

indicated. Good agreement is obtained with the same 7/3 ratio, used

for $\pi + p(\pi^-, \pi^-)$. The dashed curve represents the double excitation

contribution which is obtained for $1.6 < y_1 < 2.4$ with π^- in a back to

back configuration.

Figure 5   Distributions for $\pi - p(\pi^-, \pi^-)$ at 18.5 GeV/c. Data from

Notre Dame (Ref. 9) are plotted for the following interval of y_1:

On the left

a) $-0.2 < y_1 < 0.2$; b) $-0.6 < y_1 < -0.2$; c) $-1.0 < y_1 < -0.6$; d) $-1.4 < y_1 < -1.0$;

e) $-1.8 < y_1 < -1.4$; f) $-2.2 < y_1 < -1.8$, and g) $-2.6 < y_1 < -2.2$. On the

right: h) $0.2 < y_1 < 0.6$; i) $0.6 < y_1 < 1.0$; j) $1.0 < y_1 < 1.4$; k) $1.4 < y_1 < 1.8$;

l) $1.8 < y_1 < 2.2$; m) $2.2 < y_1 < 2.6$ and n) $2.6 < y_1 < 3.0$.

Theoretical curves are drawn for y_1 = 0.4, 1.2, 1.6, 2.0, 2.4

and 2.8 on the right for y_1 = 0. , -0.8, -1.2, -1.6, -2.0 and -2.4

on the left.

Discrepancies can be related to charge exchange effects as

already evident on Fig. 1. Normalization as well as shapes are

predictions of the model.

Figure 6   Associated multiplicity, as defined in the text, computed

for π^- production in $\pi + p$ collisions at 18.5 GeV/c. The curve is

a prediction of the model. Data points are from Ref. 9. The

agreement thus met contributes to show that the clustering

effect, already inferred from the analysis of single particle

distributions, is enough to predict the observed correlation effects.

Figure 7   The correlation function $C(y_1, y_2)$ as defined in the text

(Eq. 2.27) plotted versus y_2 for the following values of y_1

y_1 = -2.0; -0.8; 0; 0.8 and 2.0. The strong positive correlation

when both rapidities are small results from our associating the

two π^- with a heavy nova.

Figure 8(a)   Mean number of π^- for inelastic pp collision as a function

of laboratory momentum.   Data are from Michigan State (Ref. 12),

Serpukhov (Ref. 43), Echo Lake (Ref. 26) and ISR (Ref. 45).   For

both Serpukhov and Echo Lake, we use $<n^-> = 0.5(<n^{ch}> -2)$

and for ISR we take $<n^-> = <n^0>$, assuming that all γ rays come

from π^0 decay.   The solid curve is a prediction of the Nova model

as described in the text.   The mean multiplicity is severely

determined by the p_T^2 and y distributions.

Figure 8(b)   Quantity $f_2 = <n^-(n^- - 1)> - <n^->^2$ as a function of p_{lab}.

Both numbers refer to inelastic collisions.   The solid curve is a

prediction of the Nova model.   Echo Lake and Serpukhov points

are derived from prong cross sections.   Were our disagreement

with Echo Lake considered serious, it would mean that the pion

yield of very large nova cluster ceases to be linearly proportional

to the excitation mass as we assumed it to be.

Figure 9   Double rapidity distribution calculated for the reaction

$K^+p(K^0, \pi^-)$ at 16 GeV/c.   Quantity $d^2\sigma/dy_\pi dy_K$ is plotted

versus y_π for four different values of y_K.   These distributions

are presented as an illustration of the trend which should be

observed, since our treatment of kaon production from a proton

nova involves extra assumptions.   The transverse motion of

the nova, no longer negligible here, should also give some extra effect on the kaon rapidity distribution.

TABLE I

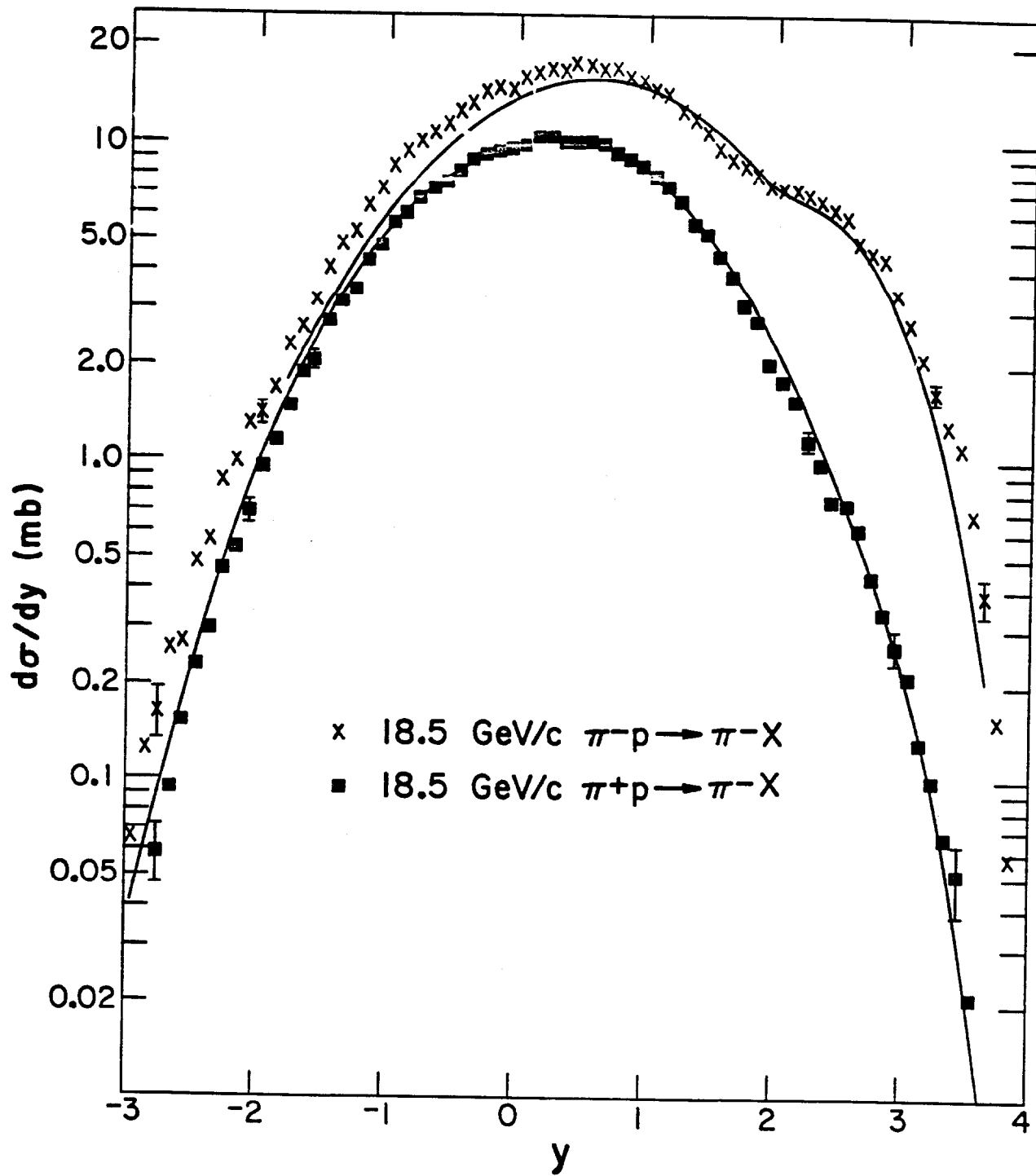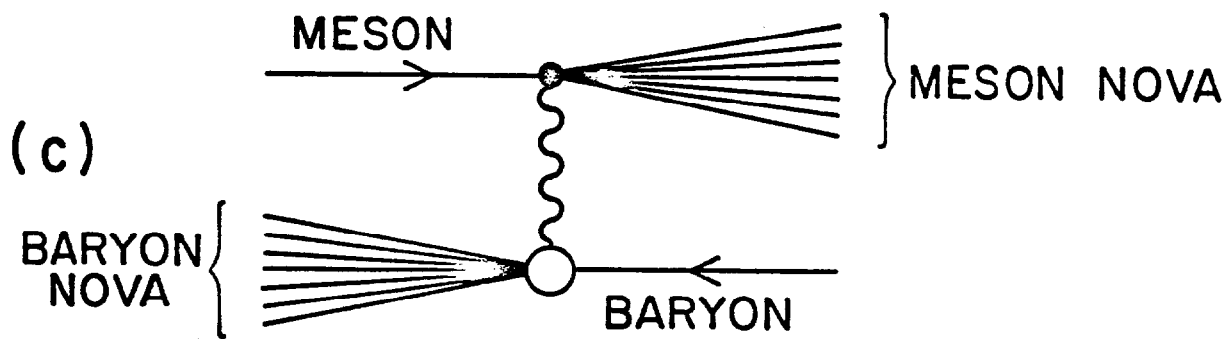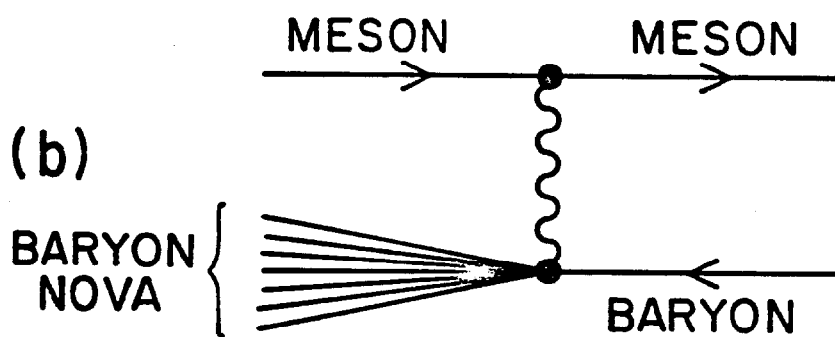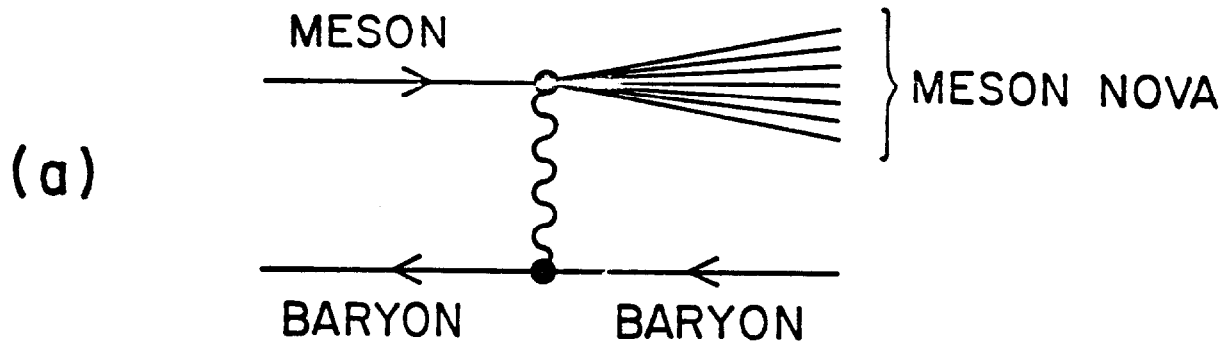|	π^+p		π^-p	
	Th	Exp	Th	Exp
$\langle n^- \rangle$	1.35	1.34±0.05	2.37	2.20±0.06
$\langle n^-(n^--1) \rangle$	1.31	1.27±0.05	3.49	3.55±0.10

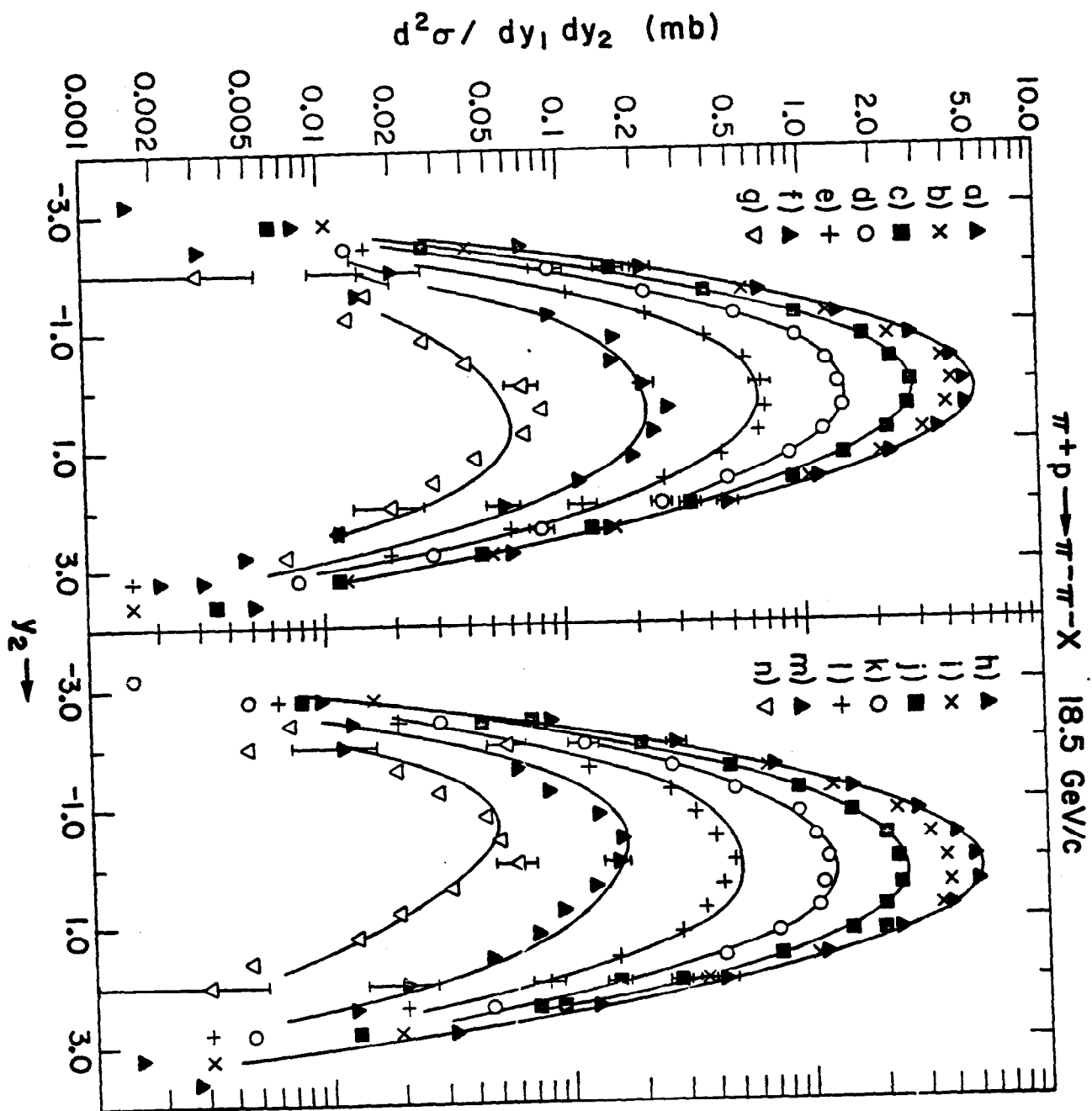Caption for Table I   Average values for $\pi^{\pm}p$ at 18.5 GeV/c. Data are from Ref. 9. All numbers are computed with respect to σ_{inel}. Specifically elastic two prong events are subtracted from σ_2 before calculation of $\langle n^- \rangle$.
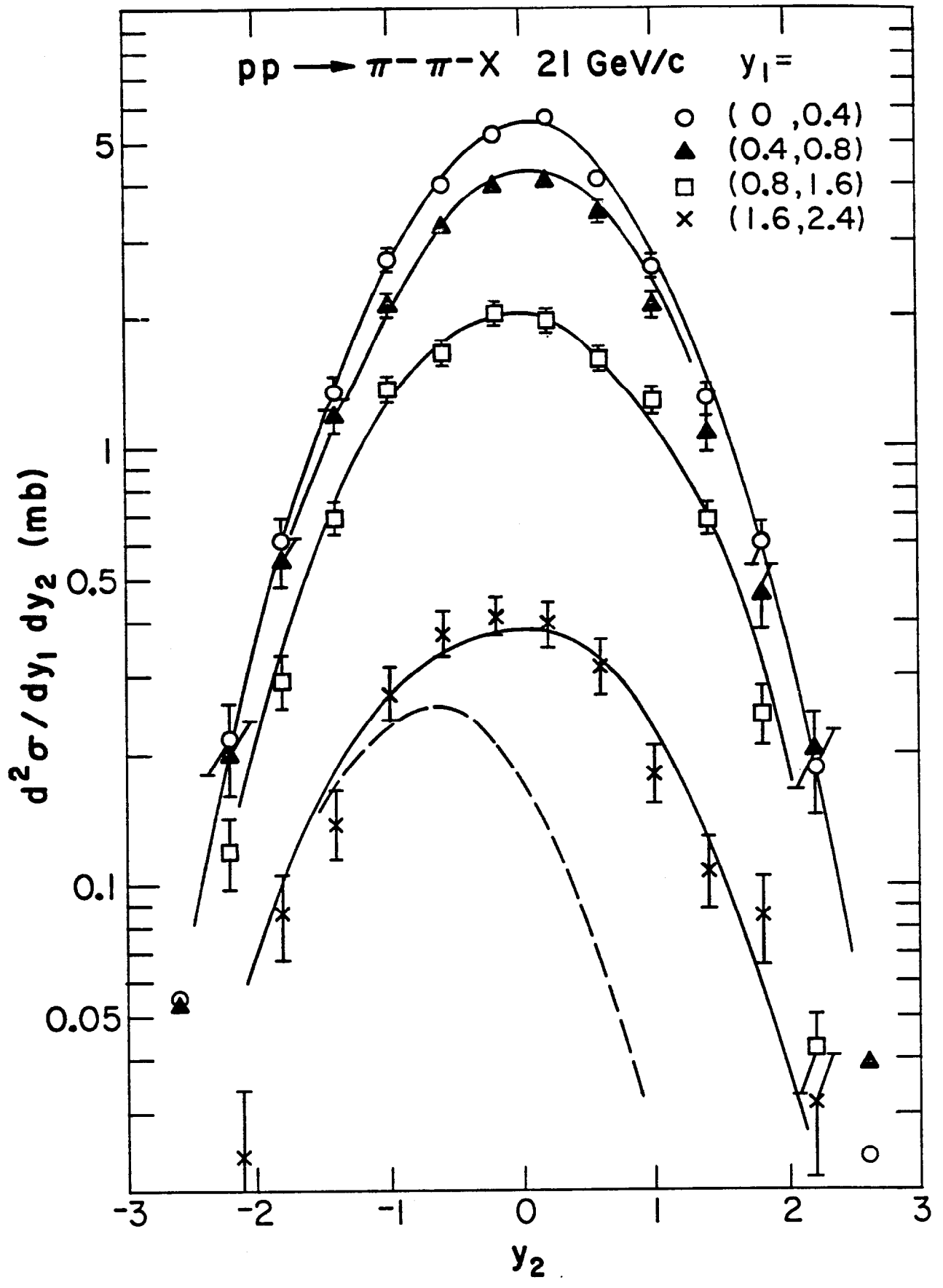
TABLE II

100 GeV/c	$\pi + p$	$\pi - p$
$\langle n^- \rangle$	2.31	3.27
$\langle n^-(n^--1) \rangle$	6.86	11.33

Figure shows dσ/dy (mb) versus y with data:
- × 18.5 GeV/c $\pi^- p \longrightarrow \pi^- X$
- ■ 18.5 GeV/c $\pi^+ p \longrightarrow \pi^- X$

(a)

MESON

MESON NOVA

BARYON   BARYON

(b)

MESON   MESON

BARYON
NOVA

BARYON

(c)

MESON

MESON NOVA

BARYON
NOVA

BARYON

$\pi^+ p \rightarrow \pi^- X$ 18.5 GeV/c

$K^+ p \longrightarrow K^o \pi^- X$ 16 GeV/c

$Y_K = 0.4$

$+1.2$

-0.4

-0.8

$\sigma_{inel}^{-1} \cdot d^2\sigma/dy_K dy_\pi$

10^{-1}

10^{-2}

10^{-3}

-2 -1 0 2

y_π