ANL/HEP 7205
NAL/THY-32

TWO PARTICLE CORRELATIONS IN INCLUSIVE REACTIONS†

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February 1, 1972

ABSTRACT

Two particle inclusive distributions and correlation functions are computed from a model which emphasizes single production of resonance-like hadrons (novas) which cascade decay through pion emission. Strong positive correlations are predicted at small values of the relative rapidity (Δy) of the final pions. The magnitude and energy dependence of $\langle n_{\pi^-} \rangle$ and $\langle n_{\pi^-}(n_{\pi^-} - 1) \rangle$ are reproduced correctly. Good agreement is obtained with experimental $d\sigma/dy$, $d^2\sigma/dy_1 dy_2$, and $C(y_1, y_2)$ from data on $\pi^+ p \rightarrow \pi^- \pi^- X$ at 18.5 GeV/c.

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† Work performed under the auspices of the U. S. Atomic Energy Commission.

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A wealth of information has become available on single particle (inclusive) distributions[1] in high energy hadronic interactions. Many theoretical points of view are consistent with present data,[2] but one may expect that differences between models will be conceded when data are confronted at higher energies. Differences between models may also be found in the analysis of correlations[3] among produced secondaries, in data at present machine energies. These correlations may represent obvious interactions among particles in the final state (e.g. resonances), but should also provide insight into the multiparticle production mechanism proper. In this paper, we focus on correlations between two π^- in two particle inclusive reactions such as $\pi p \to \pi^- \pi^- X$ and $pp \to \pi^- \pi^- X$. Because the $(\pi^- \pi^-)$ system has exotic quantum numbers, we expect that any correlations will reveal information primarily about the production mechanism.

Correlations will appear among the variables describing the final particles, e.g. the rapidities y, the transverse momenta p_T, and the angle φ between transverse momentum vectors. In $K^+ p \to \pi^- \pi^- X$, a maximum is observed in the two pion distribution at zero relative rapidity.[4] This may suggest a physical picture in which both secondaries are fragments of the same object (beam or target). It is tempting to pursue this picture in analyzing more extensive data available now from pion induced reactions.[5] In view of the large number of variables, it is hard to reach conclusions at a general level. Moreover, because it is valuable to have some preconceptions when examining new data, we adopt the quantitative framework offered by the nova model of particle production.[6] In this (fragmentation) model, single excitation of either the beam or the target particle is followed by eventual cascade decay of the resonance-like excited hadron. Although single excitation is not exhaustive, the model reproduces simply all important features of single particle distributions at current accelerator energies. Well defined predictions are made also for two particle correlations, which, as we will show, are in reasonable agreement with available data. Our results, displayed in Figs. 1, 2 and 3, represent the first realistic predictions for two particle correlations at presently available energies. Several experimental analyses are now in progress,[5, 7, 8] and our detailed model calculations may be of value.

Predicted single-particle distributions should reproduce correctly the observed rapidity and transverse momentum behavior of spectra, and be normalized to $<n> \sigma_{inel}$. Even if the total inelastic cross section σ_{inel} is kept as a normalization constant, as in our case, the fractional number of secondaries of a given charge should be a prediction of the model. When analyzing two particle spectra, one should reproduce a five dimensional distribution, $d^5\sigma/dy_1 dy_2 dp_{T1} dp_{T2} d\varphi$, whose integral is $<n(n-1)> \sigma_{inel}$. The average value $<n(n-1)>$ should also be a prediction of the model.

As discussed in detail in Ref. 6, upon summing contributions from excitation of beam and target, we write the single particle rapidity distribution as

$$\frac{d\sigma^i}{dy} = \sum_\alpha c_\alpha \int \rho_\alpha(M)\, n_\alpha^{\,i}(M)\, A_\alpha(M, y)\, dM \quad , \tag{1}$$

where α refers to either target or beam, and i labels the type of observed secondary. The integral extends over the entire mass spectrum. As written, Eq. (1) implies an average over all p_T^2. Function $A(M, y)$ is the normalized decay distribution (in y) of a nova of mass M. To obtain $A(M, y)$, we begin with the following simple symmetric decay distribution in the nova rest system:

$$dD/dp_T^2 dp_L = (a/K^2\omega) \exp[-(p_T^2 + p_L^2)/K^2] \quad . \qquad (2)$$

Here, $\omega = [p_T^2 + p_L^2 + m_\pi^2]^{\frac{1}{2}}$; a normalizes the distribution to unity; and K (~ 0.45 GeV/c) is adjusted to reproduce a typical Q value of 330 MeV at each step in the decay chain of the nova.[9] This value of K also leads to good agreement with distributions in p_T. Function $A(M, y)$ is obtained after Lorentz-transforming Eq. (2) to the center of mass system, and performing an integral over p_T^2.

In Eq. (1), $\rho(M)$ prescribes the weight assigned to the nova of mass M; as discussed in Ref. 6, there is little freedom in the choice of this function. Function $n^i(M)$ is the average number of secondaries of a particular kind (i) expected from the decay of a nova of mass M. To obtain $n^i(M)$, we modify a statistical distribution by strong charge-related effects met in the first few steps of the decay.[6] As fixed by our average Q value, the total number $n(M)$ of pions of all charges is $2.1\,(M - M_o)$, where M_o is the mass of the excited particle (target or projectile). For a positively charged nova of mass M, we find that simple and reasonable approximations are[10]

$$n^-(M) \approx [n(M) - 1]/3 \qquad (3)$$

$$n^-(M)\,(n^-(M) - 1) \approx (n(M) - 1)(n(M) - 3)/9 \quad . \qquad (4)$$

The value zero is used when $n(M) < 1$ or 3, respectively.

This is an oversimplified picture, of course, and it is important to verify that our predicted average multiplicity reproduces the energy dependence and actual value of $<n^- >$ for $pp \to \pi^- X$. Upon combining Eqs. (3) and (4) with an excitation spectrum[6]

$$\rho(M) = (M - M_o)^{-2} \exp(-2/(M - M_o)) , \tag{5}$$

we obtain results shown in Fig. 1. Given the agreement we achieve with $<n^- >$, it would be valuable to check our predictions for $<n^-(n^- - 1)>$. It may also be noted in Fig. 1 that our calculated values of $<n^- >^2$ and $<n^-(n^- - 1)>$ are approximately equal over the energy range 20 to 30 GeV/c; thus, our results approximate a Poisson distribution in this energy range. However, as p_{lab} increases, we expect that $<n^-(n^- - 1)>$ will become decidedly larger than $<n^- >^2$. These features are in good agreement with a recent compilation of available data.[11]

Identical computations of $<n>$ and $<n(n - 1)>$ may be done for $\pi^+ p$ induced reactions. At 18.5 GeV/c, where data are available, we find $<n^- > = 1.21$ and $<n^-(n^- - 1)> = 1.30$; these may be compared with experimental values[12] 1.32 and 1.26, respectively. Although agreement is reached only at the 10% level, it should be stressed that the same model is used to describe meson and baryon excitation and decay.[13] It is instructive that $<n(n - 1)>/<n>$ is somewhat too large. This corresponds in the model to too strong a weight being assigned to the large M end of the excitation spectrum $\rho(M)$. This end is, of course, poorly known a priori. The lesson is that data on correlations probe high mass excitations, not tested in a sensitive way by single particle spectra at present energies.[14] New data on energy dependence of $\langle n(n-1)\rangle / \langle n \rangle^2$ would be particularly valuable and will provide a good test of the fragmentation hypothesis.[15] Keeping Eq. (5) as our best present guess, we are led to two particle distributions whose normalization will be too large by $\sim$ 13%. Perfect agreement can be reached at the cost of as yet ad hoc modifications to $\rho(M)$.

Attaching both observed π^-'s to the same nova, the two particle inclusive distribution for $\pi^+ p \to \pi^- \pi^- X$ is

$$d^2\sigma/dy_1 dy_2 = \sum_a c_a \int \rho_a(M)\, n^-(M)(n^-(M)-1)\, A_a(M, y_1)\, A_a(M, y_2) dM. \quad (6)$$

Again, the distribution has been averaged over transverse momenta, but the simple form of Eq. (2) allows easy determination of p_T dependences.

Results obtained from numerical evaluation of Eq. (6) are shown in Fig. 2, along with some recent data.[5, 12] The shapes of these distributions, which emerge from our essentially zero parameter model, are in reasonable agreement with data, especially at small y where the cross-section is largest. This shows that the two π^- have a strong tendency to follow each other, a feature stressed by the model. Discrepancies between our calculations and the data appear systematically at larger y, where cross-sections have dropped by an order of magnitude. This is not surprising because double-excitation (e.g. production of ρN^*), is not present in our model and is likely to give back-to-back rapid π^- secondaries. At intermediate values of y, experimental distributions are broader than our computations. This effect is connected to the overly simple form of Eq. (2) which neglects polarization effects and falls too sharply at large p_T. Although our picture does not accommodate all observed features, Fig. 2 shows that it seems to be capable of reproducing the bulk of present data. Our model thus provides dominant effects from which corrections can be contemplated.

Two particle distributions in pp and Kp reactions are readily calculable and show similar treads. It will be very interesting to compare them with forthcoming data in the 10-30 GeV/c range.[5,7,8]

To summarize effects such as those shown in Fig. 2, one can define a correlation function $C(y_1, y_2)$ as follows:

$$C(y_1, y_2) = \frac{1}{< n(n-1) > \sigma_{inel}} \frac{d^2\sigma}{dy_1 dy_2} - \left(\frac{1}{< n > \sigma_{inel}}\right)^2 \frac{d\sigma}{dy_1} \frac{d\sigma}{dy_2} . \qquad (7)$$

This definition differs slightly from others found in the literature,[2] but seems more appropriate in an energy regime where $< n >$ is of order unity. Observe that Eq. (7) agrees with more common definitions if the distribution in n obeys a Poisson law. The integral $\int C(y_1, y_2) dy_1 dy_2 = 0$. In Fig. 3, we give values of C computed for several values of rapidity. The pronounced structure is remarkable. The fact that correlations are indeed large may be appreciated from the observation that the ratio $C/[\sigma_{inel}^{-1} < n(n-1) >^{-1} d^2\sigma/dy_1 dy_2] = 0.4$ at $y_1 = y_2 = 0$. Since we keep only dominant single excitation terms, this may be an overestimate.

As mentioned previously, correlations in y for different values of p_T are calculated easily. An interesting effect is that the two particle rapidity distributions become broader when p_T is restricted to be small.

This effect reflects our sequential decay picture, as expressed by the statistical distribution, Eq. (2). A finer test of the notion of sequential decay is the weakness of any correlation in φ for two π^-, which are rarely emitted in successive steps.[7] Stronger back-to-back φ correlations can be present for $\pi^+\pi^-$. As a final point, we remark that our analysis proceeds in slightly different way for inclusive processes in which one of the observed final particles has quantum numbers identical to one of the incident particles. Another term must be added to Eqs. (1) and (6), corresponding to the quasi-elastically scattered primary. The relations given in Eqs. (3) and (4) are also modified. As a result, $d^2\sigma/dy_1dy_2$ at small y is approximately three times as large for $\pi^-p \rightarrow \pi^-\pi^-X$ as for $\pi^+p \rightarrow \pi^-\pi^-X$ at 18.5 GeV/c. These points will be discussed elsewhere

We are indebted to Dr. W. Shephard and Mr. J. Powers of Notre Dame for generous access to data, and we are grateful to Dr. R. Slansky (Yale) and Dr. G. Thomas (Argonne) for valuable discussions.

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10. Oversimplifications are involved in attaching fixed pion multiplicity to each excitation mass and in calculating relative weights for each charge configuration. Nevertheless, only small corrections result from including other possibilities, such as double steps in the decay chain.

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13. The only extra input is the relative value of the constants c which are defined through Eq. (1). We take $c_\pi/c_p = 2$, which reproduces the shape of the single particle spectrum.

14. In a model independent way, the character of distributions for small y can be probed by studying single particle distributions at very high energy or, at fixed energy, by studying higher and higher correlations, which obtain their weight at smaller and smaller y.

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FIGURE CAPTIONS

1. Presented are calculated values of $<n>$, $<n>^2$, and $<n(n-1)>$ for pp reactions. Here, n stands for the number of π^-. The mean is defined with respect to σ_{inel} (e.g. $<n> = \Sigma\, n\, \sigma_n/\sigma_{inel}$). Data shown are taken from Ref. 16; experimental points have been scaled to correct for the fact that they are defined with respect to σ_{tot}.

2. Calculated distributions $d^2\sigma/dy_1dy_2$ are compared with Notre Dame data on $\pi^+p \to \pi^-\pi^-X$ at 18.5 GeV/c, Refs. 5 and 12; y_1 and y_2 are rapidities of the two π^-'s. Theoretical results are normalized to data at $y_1 = y_2 = 0$ because, as discussed in the text, our prediction for $<n(n-1)>$ is ~ 13% too high. Curves are computed at fixed values of y_1 indicated on the figure, whereas data are averaged over a band $\Delta y = 0.4$ centered at values shown. The relatively stable maximum at $y = 0.4$ corresponds to a nova mass of ~ 4 GeV, which is reasonable for production of at least two π's.

3. Correlation function $C(y_1, y_2)$ for $\pi^+p \to \pi^-\pi^-X$ at 18.5 GeV/c is plotted versus y_2 for various values of y_1. See Eq. (7) of text for definition of C.

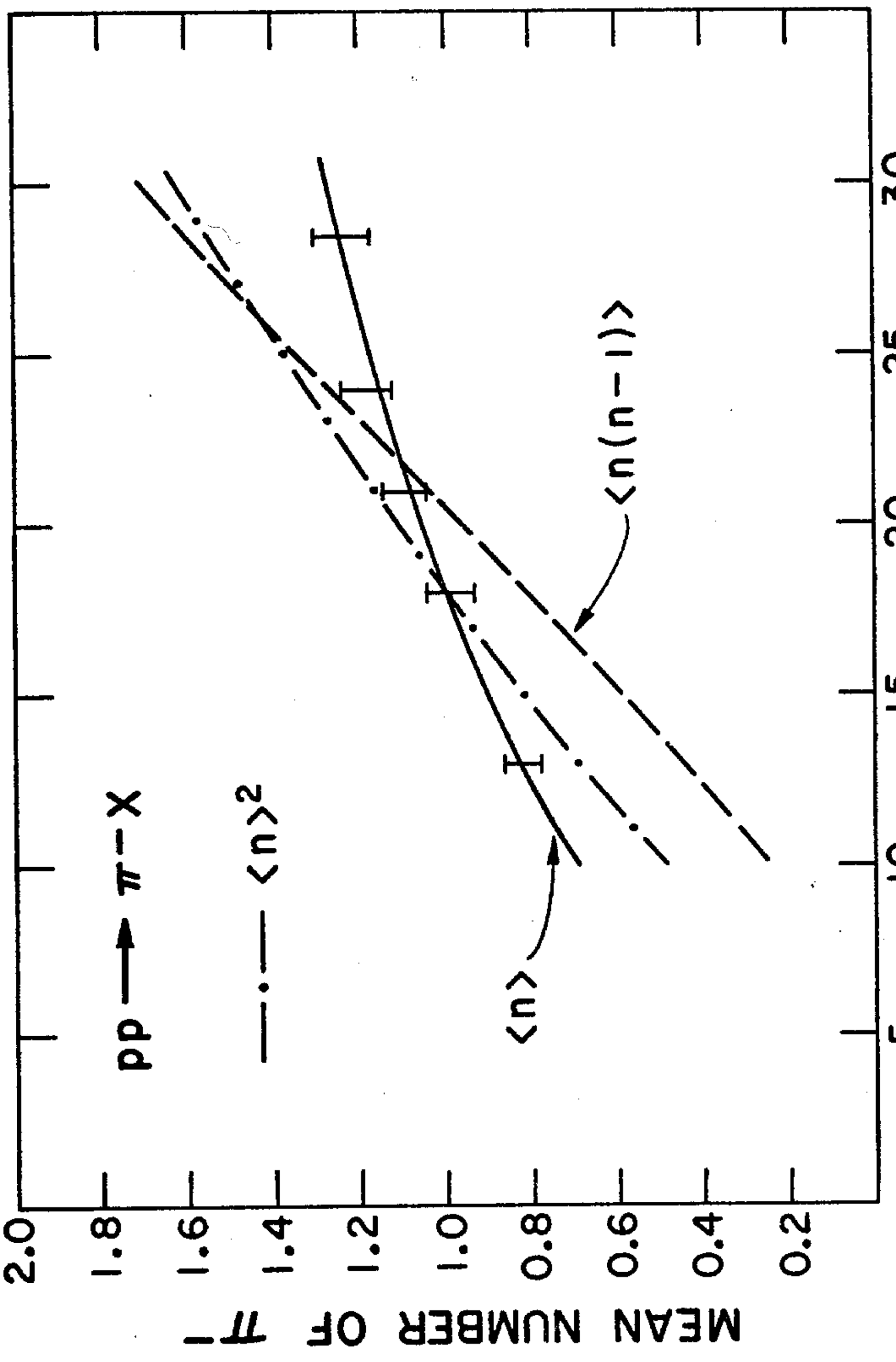
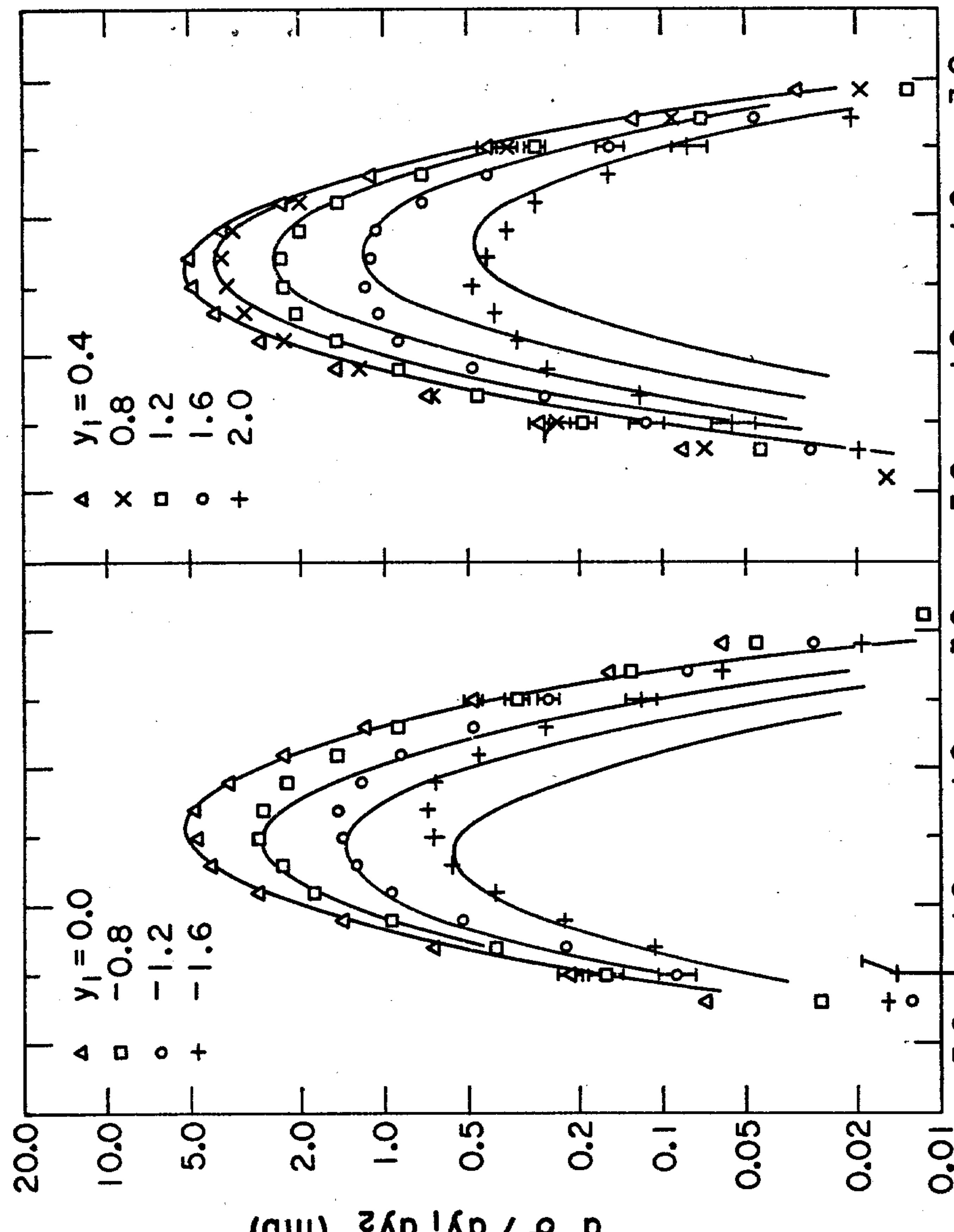
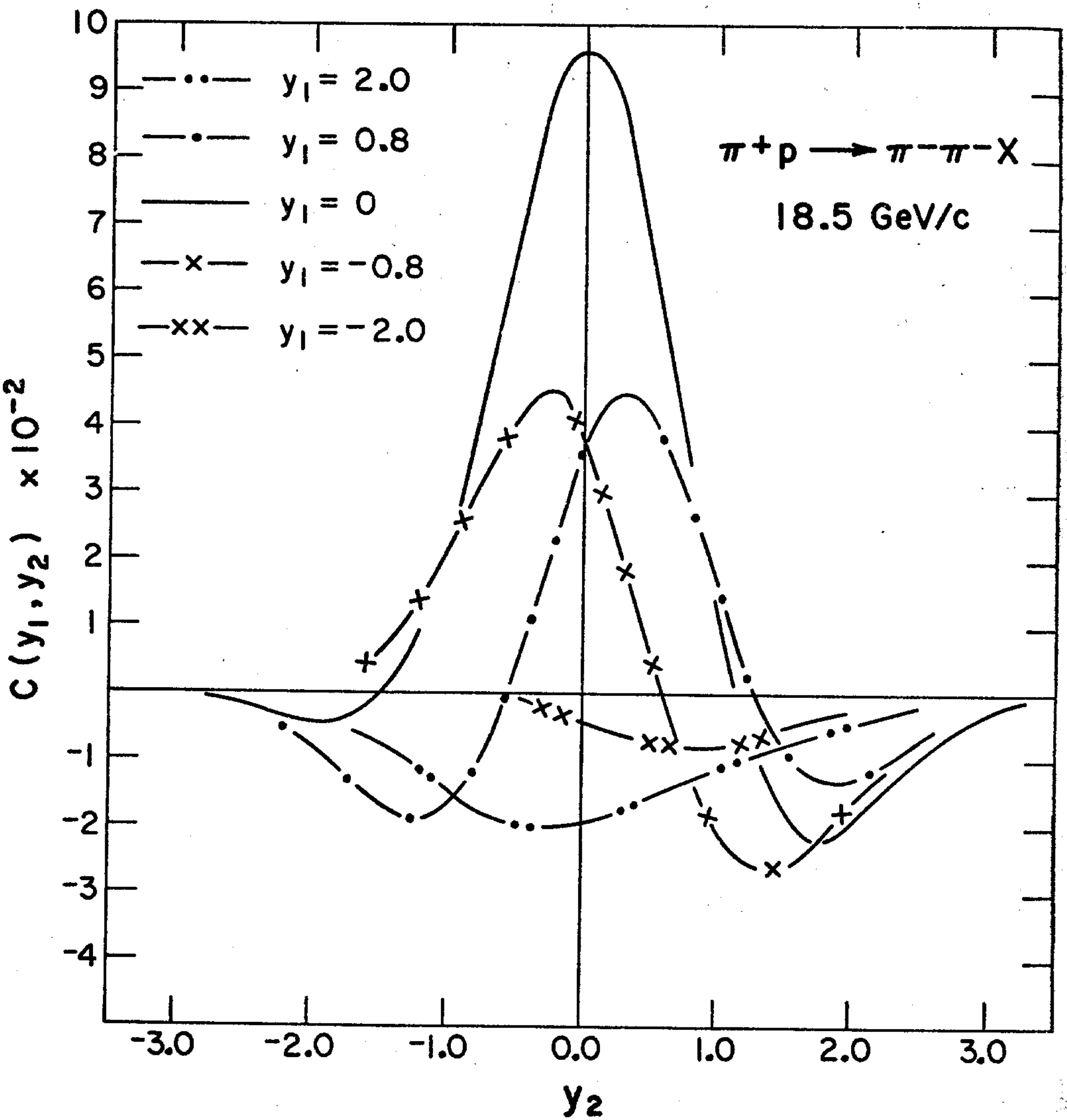

pp → π⁻ X
—·— ⟨n⟩²
⟨n⟩
⟨n(n−1)⟩
P_LAB
5
10
15
20
25
30
MEAN NUMBER OF π⁻
2.0
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2

Fig. 1



$y_1 = 0.0$
-0.8
-1.2
-1.6
$y_1 = 0.4$
0.8
1.2
1.6
2.0
y_2
-3.0
-1.0
1.0
3.0
20.0
10.0
5.0
2.0
1.0
0.5
0.2
0.1
0.05
0.02
0.01
$d^2\sigma / dy_1 dy_2$ (mb)

Fig. 2



y1 = 2.0
y1 = 0.8
y1 = 0
y1 = −0.8
y1 = −2.0
π+p → π−π−X
18.5 GeV/c
C(y1, y2) × 10−2
10
9
8
7
6
5
4
3
2
1
-1
-2
-3
-4
-3.0
-2.0
-1.0
0.0
1.0
2.0
3.0
y2

Fig. 3