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PHENOMENOLOGY OF THE TRIPLE-REGGE REGION

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Phenomenology Of The Triple-Regge Region

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ABSTRACT

A two term triple-Regge formula, which follows from a simple theoretical picture of the single particle inclusive cross-section, is presented and studied. A comparison is made with recent data for $pp \to p$ + anything, and $p\pi^- \to p$ + anything.

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In the two body reaction $a + b \to c + d$ there are two independent kinematic variables, say $s = (p_a + p_b)^2$ and $t = (p_a - p_c)^2$. For fixed t and large s, it has proved to be both possible and instructive to describe this process in terms of the exchange of a few Regge poles.* It has recently been suggested[1] that a generalized Regge analysis will also be useful as a framework in which to study inclusive reactions $a + b \to c$ + anything. In this case there are three independent variables, say $s = (p_a + p_b)^2$, $t = (p_a - p_c)^2$ and $M^2 = (p_a + p_b - p_c)^2$.

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*We include in this group of exchanges the Pomeron or vacuum exchange although its detailed character is still in doubt.

The essential feature of this generalized Regge analysis is that the Regge trajectories which appear are the same ones which were previously studied in two body reactions. Thus we have an entirely new region of physics in which to test our ideas about Regge poles and related objects, in particular the Pomeron. In this paper we shall report on an attempt to establish the validity of this generalized Regge picture by direct comparison with data. We shall focus our attention on the triple-Regge (TR) limit where t is fixed and both M^2 and s/M^2 are large. In this limit we expect the appropriate differential cross-section to be given by:

$$\frac{d^2\sigma}{dt\,dM^2} \underset{\substack{s/M^2\to\infty \\ M^2\to\infty}}{\simeq} m_0^2/16\pi \sum_{ijk} \frac{G_{ijk}(t)}{s^2} \left(\frac{s}{M^2}\right)^{\alpha_i(t)+\alpha_j(t)} \left(\frac{M^2}{m_0^2}\right)^{\alpha_k(0)} \tag{1}$$

The "α_i"s are the usual trajectory functions. The TR residue, G_{ijk}, is a product of three particle-particle-Reggeon couplings, β, and the triple-Reggeon coupling, g_{ijk}, i.e. $G_{ijk}(t) = \beta_{iac}(t)\ \beta_{jac}(t)\ \beta_{kbb}(0)\ g_{ijk}(t, t, 0)$.

Regge analysis is useful for studying two body reactions because only a few exchanges are required in order to describe the essential features of two body scattering data. To demonstrate the existence of a similar role for the TR formula [Eq. (1)] in the case of inclusive processes, it is necessary to show that again only a few terms in the infinite sum are required to adequately describe the data. It is our purpose here to attempt such a demonstration. Consideration of the general features of the data plus limited theoretical input leads to the suggestion that at least two terms are necessary in Eq. (1). We shall show by comparison with experiment that these two terms seem also to be sufficient to describe the essential features of the existing data. Further, the agreement holds over an unexpectedly large kinematical region. We caution the reader, however, that the present study is not intended as a precise fit to the data, but rather as an initial test of the basic triple-Regge picture.

Let us review the general features of the data. We will limit ourselves to the case $a = c$ so that vacuum quantum exchange is possible in the $a\bar{c}$ channel. In particular, we have studied the data for $pp \rightarrow p + X^+$ [2,3], and $p\pi^- \rightarrow p + X^-$ [4] (here and below the symbol X stands for "anything").

The data show three main features: 1) For low M^2 ($M^2 \lesssim 4\ \mathrm{GeV}^2$) there is resonance structure in M^2 and the production cross-section for these resonances seem to be independent of s; 2) In this same M^2 region there is a background contribution which seems to behave essentially as $1/s$; 3) As a function of M^2, $d\sigma/dtdM^2$ is first decreasing just above the resonances and then starts to rise for $M^2 \gtrsim 10\ \mathrm{GeV}^2$.

This behavior can be easily interpreted in terms of the usual Regge ideas. We say that a) the resonances are being produced via Pomeron exchange; b) the background results from the usual Reggeized meson ($\rho f A_2 \omega$) exchange. Thus we expect at least the two contributions pictured in Fig. 1(a).

The question still remains as to how to describe these contributions in the language of the triple-Regge formalism. In principle this is a question which can be answered by the data as more becomes available. However, at the present time we shall use theoretical input in order to arrive immediately at a unique, simple answer. This we may then compare to the data. Specifically, we shall apply duality in the form of the Freund-Harari[5] conjecture to Fig. 1(b). We interpret this to mean that the resonances are dual to the appropriate combination of ordinary meson trajectory exchanges (henceforth labeled simply as f) and the background to Pomeron exchange. Hence we are led to try to describe the data in terms of the diagrams shown in Fig. 1(c). We shall call them the PPf and ffP contributions respectively.[6] It must be pointed out, however, that, in principle, such a description is only expected to work over a very limited range of M^2. The major point of this paper is that with only these two terms all the essential features of the data are well described over a large range of M^2, in fact, from M^2 in the resonance region ($\propto$ 3 GeV^2), up to M^2 being a sizeable fraction ($\sim 1/4$) of s.

Limiting the summation of Eq. (1) to the two terms discussed, we have*

$$\frac{d^2\sigma}{dt\,dM^2} \cong \frac{m_0^2}{16\pi s^2}\left[G_{PPf}(t)\left(\frac{s}{M^2}\right)^{2\alpha_P(t)}\left(\frac{M^2}{m_0^2}\right)^{\alpha_f(0)} + G_{ffP}(t)\left(\frac{s}{M^2}\right)^{2\alpha_f(t)}\left(\frac{M^2}{m_0^2}\right)^{\alpha_P(0)}\right] \quad (2)$$

In what follows we shall use $\alpha_p(t) = 1.0 + 0.5\ (\mathrm{GeV})^{-2}t$, $\alpha_f(t) = 0.5 + 1.0\ (\mathrm{GeV})^{-2}t$, and $m_0^2 = 1.0\ \mathrm{GeV}^2$ for definiteness.** Having written down Eq. (2), we can divorce ourselves from any specific theoretical picture and ask simply whether or not it adequately

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*There is a technical question about the variable M^2 in Eq. (2) if one wants to continue it down to $M^2 \lesssim 6\ \mathrm{GeV}^2$ as for example in conjunction with a finite energy sum rule. With this in mind the calculations presented in this paper correspond to replacing M^2 by $\overline{M}^2 = M^2 - t - m_b^2$, a symmetrical variable much like the well known $\nu = \frac{s-u}{2m}$ of πN scattering. This change plays an important role only in the resonance region. In principle this question also arises for the variable s when s/M^2 is not too large (i.e., $s/M^2 \lesssim 4$) but such effects have not been included here.

**We have also studied the case $\alpha_P = 1$, independent of t. The results are quite similar to those appearing in the text except the shape of the theoretical curve at small M^2 ($M^2 \lesssim 5\ \mathrm{GeV}^2$) is now t independent and will be much larger than the data at large t.

describes the data. It is important to note that this simple two term TR formula makes very strong predictions. At a fixed value of t, it specifies both the M^2 and s dependence of the cross-section in terms of just two constants (G_{PPf} and G_{ffP}).

In order to test all of these features of our simple TR formula, we have studied the reaction $pp \to p + X^+$ at two values of s[2,3] and many values of t ($.15\ GeV^2 \leq |t| \leq 1.5\ GeV^2$) for which data are available over a wide range in M^2. In order to discuss all these data we chose specific forms for G_{PPf} and G_{ffP} as functions of t. In the absense of a complete theory, we have chosen the simplest reasonable parameterizations of these couplings consistent with a preliminary study of the data. Using $G_{PPf}(t) = 2.2 \times 10^2\ e^{.6t}\ mb/GeV^2$ (t in GeV^2) and $G_{ffP}(t) = 1.6 \times 10^3\ mb/GeV^2$ * we have evaluated Eq. (2) and compared it to the data in Fig. (2). One observes remarkable agreement over a large region of phase space.

To further test the TR picture, we have exploited the property of factorization, i.e. the fact that $G_{PPf}(t) = \beta^2_{Paa}(t)\,\beta_{fbb}(0)\,g_{PPf}(t)$ and $G_{ffP}(t) = \beta^2_{faa}(t)\,\beta_{Pbb}(0)\,g_{ffP}(t)$. Our procedure was to utilize our

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*The coupling $G_{ffP}(t)$ does not, in fact, seem to be independent of t nor does it seem to be a simple exponential. Since the variation is limited over our range of t, we were able to simply represent it as a constant and still allow a reasonable comparison with the data.

results for $pp \to p + X$ to predict the cross-section for $\bar{p}\pi \to p + X$. This was possible since the only further inputs needed to convert our two term TR formula for the pp reaction to the one for $p\pi$ were the ratios $\beta^{(0)}_{f\pi\pi} / \beta^{(0)}_{fPP}$ and $\beta^{(0)}_{P\pi\pi} / \beta^{(0)}_{Ppp}$. Having no well established, specific values for these ratios and in keeping with the qualitative nature of the present work, we have taken both ratios to be 2/3, as in a naive quark model and in reasonable agreement with two body scattering data. Comparison with the new results of Ref. (4) are shown in Fig. (3) where the individual PPf and ffP contributions are explicitly indicated. Since the data included a finite range of t we have performed an integral over t using the specific forms of the G's given above. Considering the uncertainty of the relative normalization of the $p\pi$ and pp data and the simplicity of the present model, we regard the prediction to be in satisfactory agreement with the data.

In summary, we have considered a simple triple-Regge picture in which only the PPf and ffP terms (see Fig. 1) make important contributions. We find the agreement of this picture, Eq. (2), with the existing data most encouraging. This agreement holds over a surprisingly large range of the variables, e.g. from M^2 as a small as 3 GeV^2 up to $M^2 \approx s/4$. It is by no means trivial that the theoretical cross-section is essentially flat in the intermediate M^2 region, but rises at both ends as does the data. The origin of this behavior lies in the characteristics of the two terms included. The PPf term decreases faster, as a function of M^2, than any other obvious TR term; whereas the ffP term* is the only obvious increasing function of M^2 (for $t < 0$). We have also noted that the success of this two term picture can be easily interpreted in terms of the general features of the data and a generalization of the Freund-Harari conjecture. It must be emphasized, however, that our analysis is intended to show only sufficiency of the TR terms PPf and ffP. Certainly one expects, at present energies, to have finite but small contributions from non-leading terms such as $\rho\rho f$, corresponding to Δ production. The more intriguing questions is the role of the theoretically interesting PPP term. In terms of s and M^2 dependence, PPP differs only by a factor of M from PPf. We have tried describing the data with only PPP and ffP. We find that in the true TR region (e.g. $M^2 > 6$ GeV^2, $s/M^2 > 6$) PPP + ffP and PPf + ffP give equally acceptable descriptions of the data. Thus, the separation of the PPP and PPf contributions purely on the basis of their large M^2 behavior

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*It is also interesting to note that the ffP term is of the appropriate form to allow continuation from the TR region into the usual scaling region($\frac{M^2}{s} \cong 1 - x$, where x is the usual Feynman variable). Such behavior was suggested by R. P. Feynman, see Ref. (7). Note that a finite PPf contribution leads to "nonscaling" behavior for x near 1.

does not seem possible at present.* Presumably this problem will be solved either by the advent of larger s, larger M^2 data or by the application of more powerful theoretical tools such as finite energy sum rules.[8] The general results of the present analysis certainly suggest that the triple-Regge formalism is a useful structure within which to study inclusive reactions over a large kinematic region. Specific details, particularly the consistency with zero triple-Pomeron contribution at all t, indicate that further theoretical and experimental studies encompassing larger ranges of s, M^2 and t will be most informative.

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*If PPP is, in fact, large and there is no large s independent background in the data, as is presently observed, then we will be led to discard the Freund-Harari conjecture as interpreted here.

FIGURE CAPTIONS

1) (a) Minimum contributions needed to describe the general features of the $pp \to p + X^+$ data.

(b) The absorptive part of the six point function which gives the contribution shown in Fig. 1(a).

(c) Triple-Regge diagrams.

2) $pp \to p + X^+$ data from Refs. (3) and (4) (only a fraction of the data is shown, in particular the elastic peak is not included). The solid line is the two term triple-Regge description with PPf + ffP only. Similar curves result from PPP + ffP only.

3) $p\pi^- \to p + X^-$ data from Ref. (5). The solid line is the two term triple-Regge description normalized to the pp data. Individual PPf and ffP contributions are shown; (a) P_L = 25 GeV/c, (b) P_L = 40 GeV/c.

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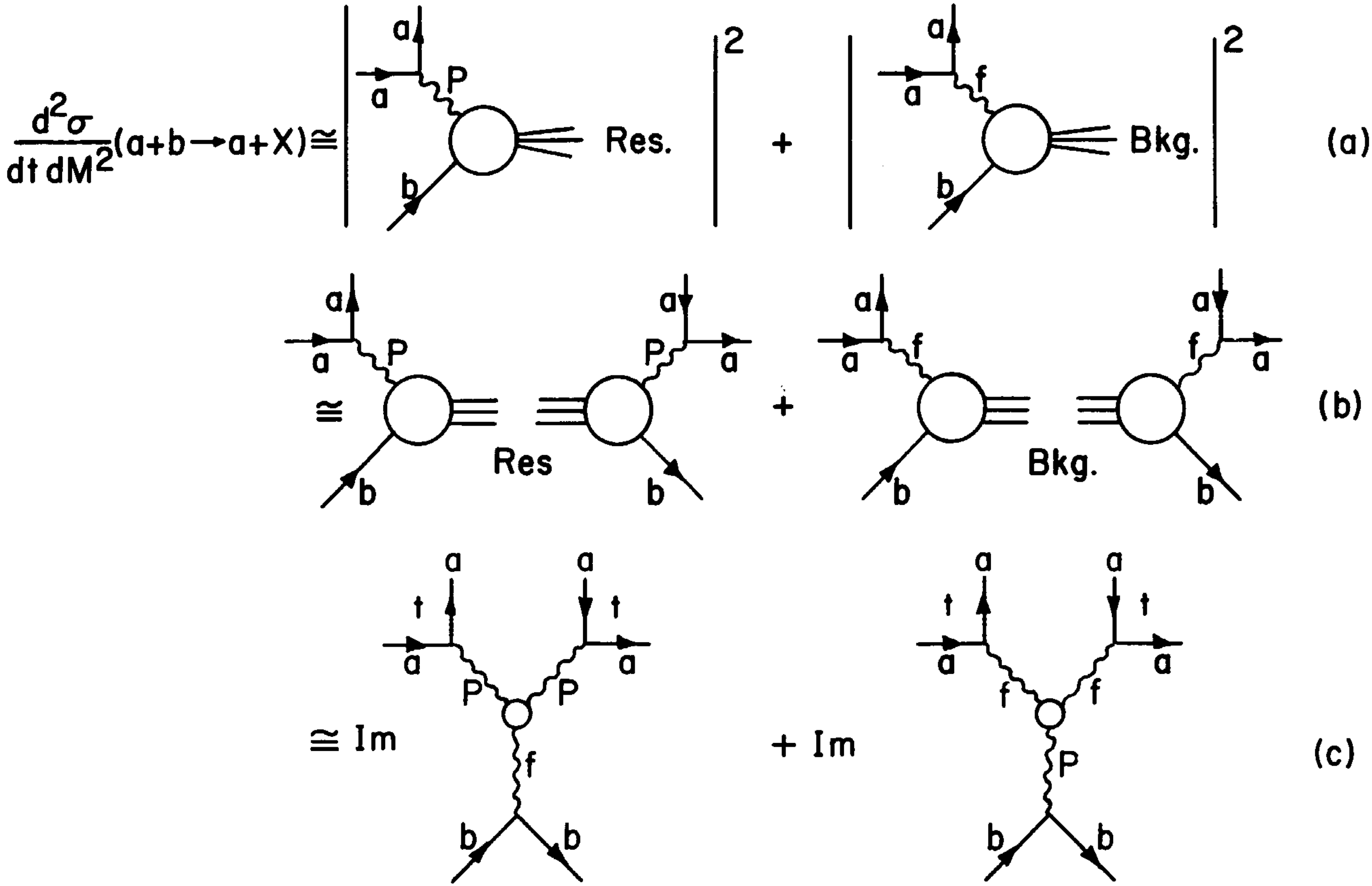
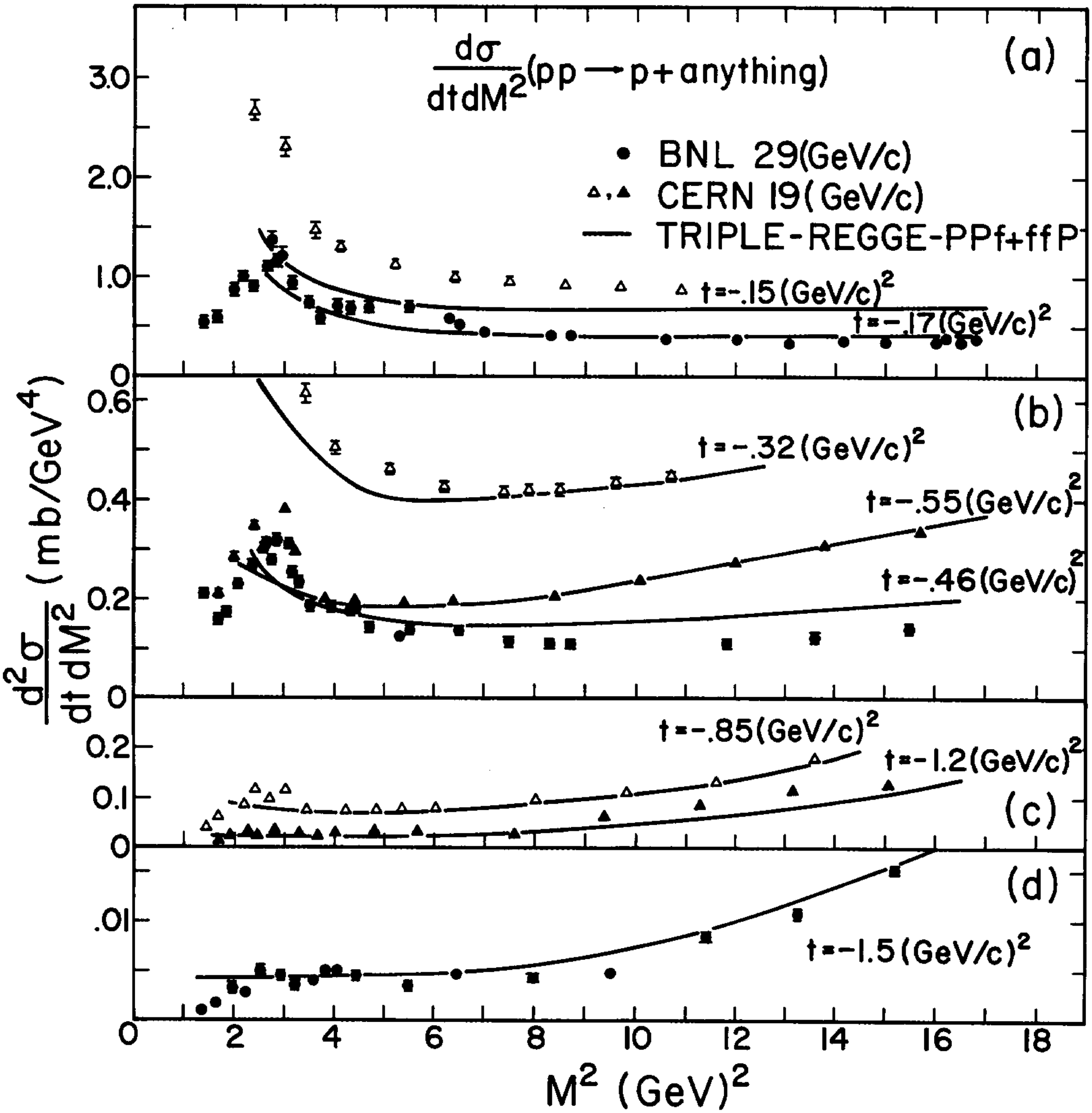
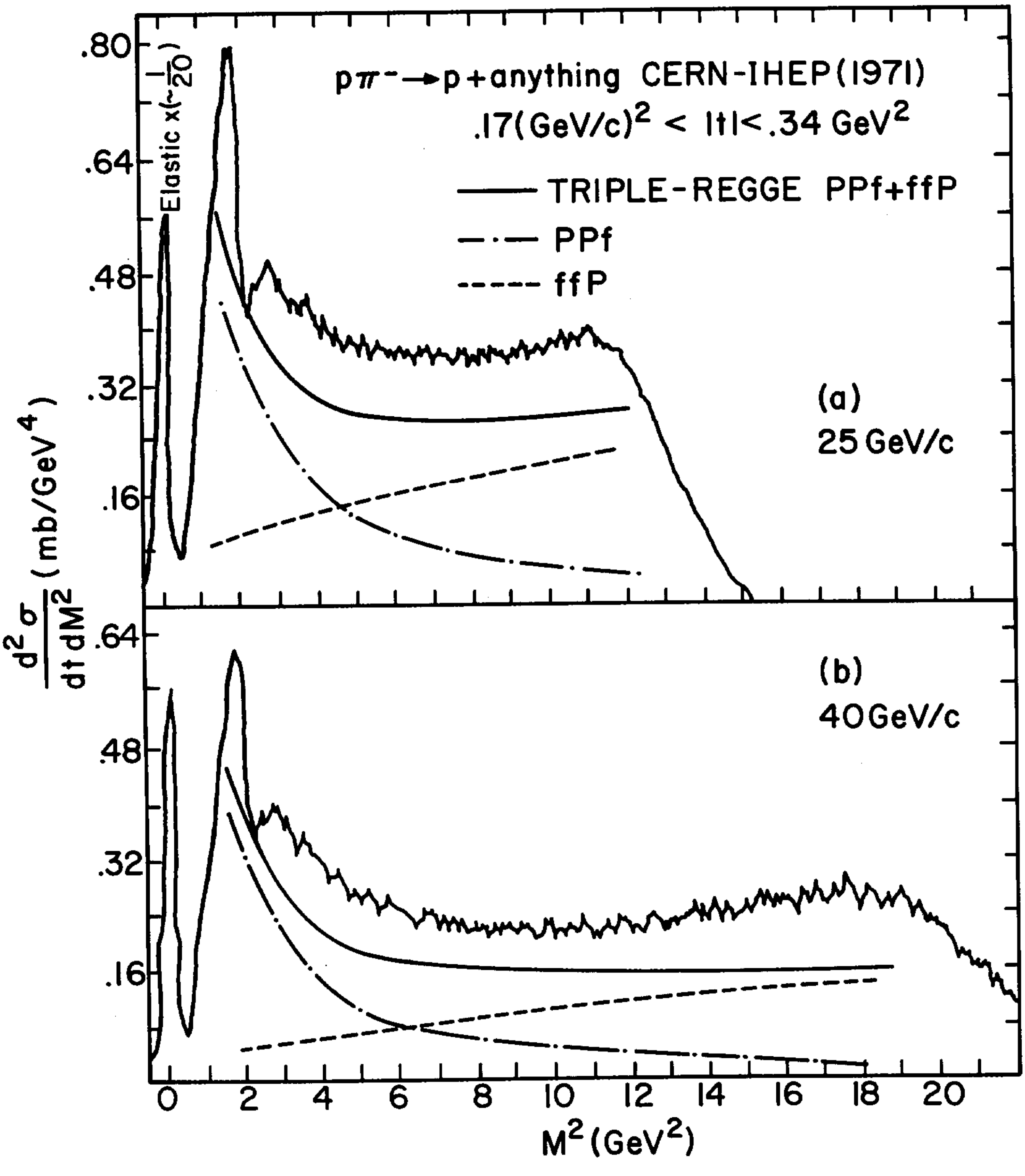

$\frac{d^2\sigma}{dt\,dM^2}(a+b\rightarrow a+X)\cong$
Res.
Bkg.
(a)
Res
Bkg.
(b)
$\cong$ Im
+ Im
(c)

Figure 1



$\frac{d\sigma}{dtdM^2}$(pp → p+anything)
(a)
BNL 29(GeV/c)
CERN 19(GeV/c)
TRIPLE-REGGE-PPf+ffP
t=-.15(GeV/c)^2
t=-.17(GeV/c)^2
(b)
t=-.32(GeV/c)^2
t=-.55(GeV/c)^2
t=-.46(GeV/c)^2
(c)
t=-.85(GeV/c)^2
t=-1.2(GeV/c)^2
(d)
t=-1.5(GeV/c)^2
$\frac{d^2\sigma}{dt\,dM^2}$ (mb/GeV^4)
M^2 (GeV)^2
3.0
2.0
1.0
0
0.6
0.4
0.2
0
0.2
0.1
0
.01
0
2
4
6
8
10
12
14
16
18

Figure 2



pπ⁻ → p + anything CERN-IHEP (1971)
.17 (GeV/c)² < |t| < .34 GeV²
TRIPLE-REGGE PPf+ffP
PPf
ffP
Elastic x(~1/20)
(a)
25 GeV/c
(b)
40 GeV/c
d²σ/dt dM² (mb/GeV⁴)
M² (GeV²)
.80
.64
.48
.32
.16
0
2
4
6
8
10
12
14
16
18
20

Figure 3