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A THEORY OF ELECTROMAGNETIC AND WEAK INTERACTIONS

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[The author's note in February 1972] The title of the talk I gave at the Princeton Conference on Local Currents was "The Role of Current Algebra in the Formulation of Broken Symmetries". I discussed the Higgs model as an illustration throughout the talk and discussed Weinberg's theory as an interesting physical application of the Higgs phenomenon.

Since then, the subject has seen a rapid and vigorous development, so much so that it seems completely out of date to reproduce my talk in writing at this time. For this reason, I submit the text of my talk at the APS San Francisco meeting on this subject. At the end of this paper, I attach a list of references distributed at the Conference for background.

I would like to describe some very intense theoretical activities in the past few months in the direction of unifying electromagnetic and weak interactions, and constructing a renormalizable theory of weak interactions. These researches are all inspired by S. Weinberg's paper in 1967[1] (and A. Salam's paper in 1968[2]), and the interest in this important paper was revived by the work of 't Hooft,[3] a young Dutch physicist, who has given a rather convincing argument for the renormalizability of the Weinberg model. Even in the original paper, Weinberg suggested that the theory might be renormalizable, and it now appears that Weinberg's conjecture was indeed correct. Whether the Almighty God in his omniscience makes use of this rather clever scheme is not entirely clear to us at the moment, however.

Weinberg's theory combines two ideas. One is to unify electromagnetic and weak interactions in a local gauge invariant theory by introducing Yang-Mills gauge bosons. The second is to break the gauge invariance spontaneously to generate the masses of vector mesons which mediate weak interactions.

The second ingredient is known as the Higgs phenomenon[4] whereby the two polarizations of a massless gauge boson combine with the Goldstone boson to produce a massive vector boson. It is a relativistic analogue of Nambu's observation[5] that in the BCS theory where electric gauge invariance is spontaneously broken, the phonon (massless particle) turns into the plasmon (massive particle). Because the Lagrangian is gauge invariant, we are at liberty to choose a gauge to quantize the theory. If we choose a relativistically invariant gauge, for example the Landau gauge, the vector boson propagator is

$$\left(g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right) \frac{1}{k^2 - m^2}$$

when the gauge symmetry is spontaneously broken.

In addition, the theory contains scalar particles (Goldstone bosons) whose propagators are

$$\frac{1}{k^2}$$

What happens ultimately is that the two poles at $k^2 = o$ cancel in the S-matrix, leaving only the massive vector bosons physical. The essence of 't Hooft's argument was that the Landau gauge vector propagator does not make the higher order amplitudes more and more divergent, but rather gives rise to the ultraviolet divergences which are at most quadratic, and that the cancellation of two zero mass poles is guaranteed by the gauge symmetry of the Lagranian.

The first ingredient of the Weinberg theory is the symmetry of weak and electromagnetic interactions. For the leptons, one can take the $SU_L(2) \times U_L(1)$ scheme first proposed by Schwinger[6] and Glashow[7], where the symmetries act on a left handed SU(2) doublet

$$L = \tfrac{1}{2}(1 + \gamma_5) \begin{pmatrix} \nu \\ \ell^- \end{pmatrix}$$

with the leptonic hypercharge $Y = -\frac{1}{2}$, and a right handed singlet

$$R = \frac{1}{2}(1 - \gamma_5)\ \ell^-$$

with the leptonic hypercharge $Y = -1$. The electric charge is given by the formula

$$Q = T_3 + Y.$$

We need four gauge bosons, two charged and two neutral, to make SU(2) x U(1) a local symmetry. In addition we need a complex scalar doublet, to break the symmetry spontaneously down to the U(1) of electric charge. This is achieved by having the neutral component of the scalar doublet develop a vacuum expectation value. The nonvanishing lepton masses are also due to this symmetry breaking mechanism.

The outcome of all this is two charged vectors $W_\mu^\pm$, a neutral massive vector Z_μ, the massless photon A_μ, associated with the unbroken charge gauge symmetry, and a massive scalar boson. The other components of the scalar doublet (the Goldstone bosons) can be completely eliminated from the Lagrangian by a suitable choice of the gauge. For lack of better names, we shall call this gauge the U gauge (unitary gauge) since in this gauge the massive vector propagator is

$$\left(g_{\mu\nu} - \frac{k_\mu k_\nu}{m^2}\right) \frac{1}{k^2 - m^2}$$

and there are no unphysical zero mass scalar bosons so that the unitarity of the S-matrix is manifest. On the other hand in the R-gauge (renormalizable gauge), the unitarity of the S-matrix hinges on the delicate cancellation of two unphysical poles.

While the renormalizability of the U-gauge theory is by no means obvious, a number of calculations exist which show that the S-matrix is finite after renormalizations. For example, Weinberg shows the quadratic divergence in various single loop diagrams in the process $\nu + \bar{\nu} \rightarrow \nu + \bar{\nu}$ cancel.[8] Applequist and Quinn have considered the same problem in a simplified model[9], and have shown that logarithmic divergences cancel also.[10] Jackiw and Weinberg have computed the $G_W m_\mu^2$ contribution to the anomalous magnetic moment of the μ-meson in this gauge.[11]

In addition to the scheme discussed above there are alternative schemes for the leptons. An example is the SU(3) x SU(3) scheme based on the Konopinski-Malamud triplet. This was discussed originally by Salam and Ward[12], and more recently by Weinberg[13] and Freund.[14] According to Weinberg, this scheme offers the possibility of understanding the electron mass in terms of the muon mass.

The scheme can be extended to include hadrons in a variety of ways. Unfortunately, the problem here is the embarrassment of rich alternatives and the seeming impossibility of incorporating the Cabibbo angle in a natural way. Perhaps the ultimate solution will come with our better understanding of the genersis of the Cabibbo angle. I will list here what I consider to be the requirements for a satisfactory theory including hadrons:

1. it must be renormalizable (at least by the power counting argument),
2. it must accommodate a nonvanishing Cabibbo angle,
3. it must contain no massless particles except the photon,
4. it should not give any strangeness changing neutral current in first order (this is a phenomenological necessity).

In addition, one may ask that the $\Delta I = \frac{1}{2}$ rule for nonleptonic processes comes out from the Lagrangian, or perhaps one may demand that no new "queer" particles be postulated.

The simplest scheme I know of, which satisfies the four conditions above, is essentially the model of Glashow, Illiopoulous, and Maiani. The strangeness changing neutral current is eliminated at the cost of postulating a fourth quark q. The unequal masses of four quarks arise from their couplings to the scalar doublet discussed previously, which must be $SU_L(2) \times U_L(1)$ invariant.[16] The symmetries act on two $SU_L(2)$ multiplets

$$\frac{(1+\gamma_5)}{2}\begin{pmatrix} p \\ n' \end{pmatrix}, \qquad \frac{(1+\gamma_5)}{2}\begin{pmatrix} q \\ \lambda' \end{pmatrix}$$

where $n' = n\cos\theta + \lambda\sin\theta$

$\lambda' = -n\sin\theta + \lambda\cos\theta$

and on four singlets, $\frac{1}{2}(1-\gamma_5)p, \ldots \frac{1}{2}(1-\gamma_5)\lambda$

This scheme has another very attractive feature. It was argued by M. Veltman[17] that the triangle anomalies discussed in recent years by Bell, Jackiw and Adler would render the theory ultimately unrenormalizable. D. Gross and Jackiw have recently given a very thorough analysis of this problem.[20] Their study confirms this, except when, as they point out, the anomalies are cancelled out internally. The model we discussed above has precisely this feature: the anomalies caused by the lepton and hadron loops cancel exactly.[21] Furthermore, this cancellation calls for the existence of the two leptonic $SU_L(2)$ multiplets, one for the electron and the other for the muon, to go along the two $SU_L(2)$ multiplets of the hadronic matter, and makes the existence of the muon compelling. This may be the answer to I. Rabi's famous question: "Who ordered the muon?".

I have been working on the renormalizability question of this theory in collaboration with Jean Zinn-Justin of Stony Brook and Saclay for the last few months,[9,22] and let me itemize here what we have been able to accomplish so far:

1. The quantization of the massless Yang-Mills theory was carried out by Feynman,[23] deWitt,[24] Mandelstam,[25] and Popov and Fadeev.[26] We show that the renormalized Green's functions exist and they satisfy the Ward identities of the theory. This is done by demonstrating that divergent Feynman integrals can be regulated in a gauge invariant manner, and therefore the BPH subtraction procedure[27] preserves the gauge invariance. The infrared divergences are avoided by a suitable subtraction procedure. These subtractions are known to be formally implementable by gauge invariant counter terms in the Lagrangian.
2. In the R-gauge, it is shown that the same counter terms as in the massless theory remove all ultraviolet divergences from Green's functions, and the renormalized Green's functions satisfy the Ward identities appropriate to the spontaneous broken symmetry case.
3. It is shown that the Ward identities imply that the S-matrix is devoid of unphysical singularities at $k^2 = 0$.
4. It is shown that the S-matrix of the R-gauge and the U-gauge are identical. This is done by expressing the U-gauge Green's functions in terms of the R-gauge ones.

Finally, let me briefly mention the phenomenological aspects of the theory. The theory, as it applies to the leptons, contains five parameters. They are g, g′, ν, m_e and m_μ. The coupling constants g and g′ appear in the interactions of the leptonic isospin and hypercharge currents with their gauge bosons and v is the vacuum expectation value of the neutral scalar field. We find that

$$e = \frac{gg'}{\sqrt{g^2+g'^2}} , \qquad \frac{G_W}{\sqrt{2}} = \frac{1}{2v^2}$$

and, with

$$m_W = \frac{1}{2} g v , \qquad m_Z = \frac{1}{2}(g^2+g'^2)^{1/2} v$$

$$g'/g = \tan\theta_W ,$$

This gives

$$\frac{G_W}{\sqrt{2}} = \frac{e^2}{8 m_W^2 \sin^2\theta} = \frac{e^2}{8 m_Z^2 \sin^2\theta \cos^2\theta} .$$

$$m_W \gtrsim 37 \text{ Gev} , \quad m_Z \gtrsim 75 \text{ Gev}$$

H. H. Chen and I have discussed the implications of the Weinberg theory on the so-called diagonal weak processes.[28] The effective Lagrangian for the process $\nu_e(\bar{\nu}_e) + e \rightarrow \nu_e(\bar{\nu}_e) + e$ is

$$\mathcal{L}_{eff} = \frac{G_W}{\sqrt{2}} [\bar{\nu}\gamma_\alpha(1+\gamma_5)\nu][\bar{e}\gamma^\alpha(C_V + C_A\gamma_5)e]$$

with

$$C_V = \frac{1}{2} + 2\sin^2\theta_W , \qquad C_A = \frac{1}{2}$$

as contrasted to the Feynman-Gell-Mann theory values $C_A = C_V = 1$. When the Reines-Gurr experiment is analysed in accordance with the Weinberg theory, one obtains the bound

$$\sin^2 \theta \leq 0.35$$

which in turn implies $m_W \geq 65$ BeV! Note that the lowest cross-section predicted by the present model is $\frac{1}{4}$ that of the Feynman-Gell-Mann theory.

For the process $\nu_\mu(\bar{\nu}_\mu) + e \rightarrow \nu_\mu(\bar{\nu}_\mu) + e$ one obtains the effective Lagrangian

$$\mathcal{L}_{eff} = -\frac{G_W}{\sqrt{2}}\left[\bar{\nu}'\gamma_\alpha(1+\gamma_5)\nu'\right]\left[\bar{e}\gamma^\alpha(C_V' + \gamma_5 C_A')e\right]$$

where $C_V' = \frac{1}{2} - 2\sin^2\theta_W$, $C_A' = \frac{1}{2}$. The Feynman-Gell-Mann theory predicts zero cross section in lowest order for this process. The cross-section expected on the Weinberg theory is somewhat less than the experimental bound as deduced by Steiner and Albright:

$$\sigma_{exp}(\nu_\mu' + e \rightarrow \nu_\mu' + e) \lesssim 0.4\, \sigma_{F.G}(\nu_e + e \rightarrow \nu_e + e)$$

for all values of $\sin\theta_W$.

For hadronic interactions such as $\nu' + p \rightarrow \nu' + p$ the theoretical uncertainty is greater. But Weinberg[29] observes that the CERN bounds

$$\sigma(\nu' + p \to \nu' + p) / \sigma(\nu' + n \to \mu^- + p) = 0.12 \pm 0.06$$

is compatible with the particular model we discussed, and with $\theta_W \leq 35^\circ$.

I thank R. Jackiw who served as a critic on the draft.

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[16] A detailed construction along this line is apparently due to S. Glashow (unpublished).

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[21] The cancellation of the anomalies arising from the electron doublet and the hadron doublet (p, n′) can be seen most easily if we write the latter as

$$\frac{1-\gamma_5}{2}\begin{pmatrix}\bar{n}' \\ p^-\end{pmatrix}$$

which should be compared to

$$\frac{1+\gamma_5}{2}\begin{pmatrix}\nu \\ e^-\end{pmatrix}$$

Both doublets have exactly the same $SU_L(2) \times U_L(1)$ quantum numbers. Since the anomalies occur only in vertices odd in the axial vector currents it is easy to see that the anomalies associated with the two doublets have opposite signs. Note further that the magnitude of the anomaly is independent of the fermion mass.

I had originally considered the possibility of adding another pair of leptons (still undiscovered) of right chirality to cancel the anomaly due to the electron doublet (Princeton Conference on Local Currents, September 1971). About two weeks before the San Francisco meeting, the possibility of the cancellation between the leptonic and hadronic anomalies occurred to me. The possibility discussed in the text (or its variations) is known also to other people; among them, S. Weinberg and P. Freund. In particular, the elegant way of writing the hadron doublet in the form (1) was first shown to me by Weinberg.

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Background Material for B.W. Lee's Talk

(distributed at Princeton Conference on Local Currents)

I would like to illustrate the role of current algebra in the formulation of (broken) symmetries with an example. The example is the Higgs model, in which a gauge boson becomes massive due to the spontaneous breakdown of gauge symmetry.

1. Goldstone Theorem. Some of the earlier discussions can be found in

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   T.W.B. Kibble, in Proceedings of the 1967 International Conference on Particles and Fields (Interscience, New York, 1967),
   G. Guralnik, C.R. Hagen and T.W.B. Kibble, in Advances in Particle Physics, Vol. II, (Interscience, New York, 1968).

2. Higgs Phenomenon

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   G. 't Hooft, Nuclear Physics B35, 167 (1971).

   For nonrelativistic analogue, see

   Y. Nambu, Phys. Rev. 117, 648 (1960),
   P.W. Anderson, Phys. Rev. 130, 439 (1963),
   A. Klein and B.W. Lee, Phys. Rev. Letters 12, 266 (1964).

   A recent discussion of this phenomenon in the context of particle physics is found in:

   S. Weinberg, Phys. Rev. Letters 19, 1264 (1967).

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B. W. Lee, Nuclear Physics, B9, 649 (1969),
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4. Effective Action Approach.

This is a formulation of (broken) symmetries in terms of the generating functional of irreducible vertices. This formalism allows the global study of low energy theorems and renormalization prescriptions.

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