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Radiation Effects Produced by Modulated Electrons*

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In the experiment of Schwarz and Hora, modulated electrons can radiate substantially more light than unmodulated electrons only if the modulated electron wave function contains many plane-wave components. The radiated light must be coherent if it is emitted from a one-electron state or collectively from a product of modulated one-electron states. These conclusions are independent of the mechanisms of excitation and radiation.

Schwarz and Hora[1] have reported that when a beam of electrons was irradiated with laser light and then allowed to strike a target, light was emitted with the laser frequency ν. It has subsequently been suggested[2] that the emitted light may be accounted for by supposing that some of the irradiated electrons acquired a modulated wave function of the form

$$\psi = \exp[i(k_0 x - \omega_0 t)] + c_+ \exp[i(k_+ x - \omega_+ t)] + c_- \exp[i(k_- x - \omega_- t)], \qquad (1)$$

where $\hbar\omega_0$ is the energy of the electron before irradiation, $\omega_\pm = \omega_0 \pm \nu$, and each wave number k obeys $k^2 = 2m\omega/\hbar$ with the appropriate frequency ω. However, the mechanism by which such a modulated electron beam might produce the required radiation effects remains in doubt.[2-4]

Some insight into this problem is obtained by examining the difference between modulated and unmodulated electrons. Given an electron beam incident upon a measuring apparatus, can this apparatus distinguish between modulated and unmodulated electrons by giving a large signal in one case and a small one in the other? We take a very general and model-independent point of view. Suppose that a beam of electrons is incident upon an unspecified black box, and that a measurement is made at a later time. We ask under what conditions the probability of observing some unspecified final state of the measuring apparatus can be much larger for a modulated electron beam than for an unmodulated beam. We rely entirely upon two principles of quantum mechanics, the superposition principle and the energy-conservation principle. We find severe restrictions from the superposition principle, which asserts that any transition probability amplitude for the case of an incident modulated electron is the sum of the transition probability amplitudes for its component plane-wave states. No large enhancement of the transition probability can occur unless the modulated state contains a large number of plane-wave components. Therefore, the three-component wave function (1) is inadequate. If the experiment measures transition rates to final states of definite energy,[5] conservation of energy additionally forbids any great enhancement of that transition probability for initial states which are products of modulated one-electron states.

Consider first a system that consists initially of a single electron and a black box, with the electron in-a plane-wave state of wave number k and with the black box in some state B. Let $A(k, B \rightarrow F;t)$ be the probability amplitude for finding the system in some final state F at a later time t. The final state may include radiation or other reaction products.

Next consider the case in which the electron is in a modulated state ψ_M that is the sum of a finite number of plane-wave states[6] φ_k, namely,

$$\psi_M = \sum_k c_k \varphi_k \tag{2}$$

$$\sum |c_k|^2 = 1. \tag{3}$$

(The φ_k and ψ_M are understood to be normalized in the same way.) Then, according to the superposition principle, the probability amplitude $A(M, B \rightarrow F;t)$ for finding the system in the final state F at a later time t is given by

$$A(M, B \rightarrow F;t) = \sum c_k A(k, B \rightarrow F;t). \tag{4}$$

The transition probabilities $W(k)$ from the plane-wave states are equal to the absolute squares of the amplitudes $A(k, B \rightarrow F;t)$. The transition probability $W(\text{mod})$ from the modulated state is given by

$$\begin{aligned} W(\text{mod}) &= |A(M, B \rightarrow F;t)|^2 \\ &= |\sum_k c_k A(k, B \rightarrow F;t)|^2. \end{aligned} \tag{5}$$

It follows from the Schwarz inequality that

$$\begin{aligned} W(\text{mod}) &\leq (\sum |c_k|^2)(\sum |A(k, B \rightarrow F;t)|^2) \\ &= \sum_k W(k). \end{aligned} \tag{6}$$

Inequality (6) implies that the enhancement of the transition probabilities is limited by

$$W(\text{mod}) / W(\text{plane}) \leq N, \tag{7}$$

where $W(\text{plane})$ is the greatest of the plane-wave transition probabilities $W(k)$, and N is the number of plane-wave components in the modulated electron wave function.

This result shows that a large enhancement can be obtained only when a large number of side bands appear in the modulated wave function of the electron. There is no possibility of explaining a large enhancement by the use of a single-electron wave function with only a few side bands, such as that of Eq. (1). This conclusion, which follows from the superposition principle alone, expresses the physical fact that a wave function that is a superposition of only a few components cannot produce large effects of constructive interference between those components. The magnitude of the effect of constructive interference is limited by the number of components.

We obtain an additional restriction by invoking conservation of energy. Suppose the initial state B of the black box and final state F of the entire system are both eigenstates of the energy; then all of the transition amplitudes $A(k, B \rightarrow F;t)$ vanish except for the one value of k that conserves the total energy. There is, therefore, only one nonvanishing term in the sum in Eq. (5), and the transition probability cannot be enhanced by modulating the initial electron wave.[7] This argument applies equally well to any statistical ensemble of initial states B that represents an incoherent mixture of energy eigenstates, and to a sum of transitions to many final states each of which is an energy eigenstate.

The physical interpretation of the result of the energy argument is also simple. Constructive interference between different plane-wave components of the modulated electron wave function is observable only in an experiment that cannot decide which component produced the final state. If the final state has definite energy, then energy conservation can be used to isolate the component that initiated the transition. Consequently, there can be no interference and no enhancement.[8]

The possibility of cooperative radiation by a few electrons does not substantially change these conclusions. Consider a system that consists initially of two electrons plus the black box. Then

$$\psi_M^{(2)} = \sum_{j,k} c_{jk} \varphi_j(1) \varphi_k(2), \tag{2'}$$

$$\sum_{j,k} |c_{jk}|^2 = 1, \tag{3'}$$

$$A_2(M, B \rightarrow F;t) = \sum_{j,k} c_{jk} A_2(j, k, B \rightarrow F;t), \tag{4'}$$

$$W_2(\text{mod}) \leq \sum_{j,k} W_2(j, k), \tag{6'}$$

$$W_2(\text{mod}) / W_2(\text{plane}) \leq N_2. \tag{7'}$$

The consequences of the superposition principle remain completely unchanged except that now the enhancement factor for the modulated electrons is limited by the number N_2 of two-electron plane-wave components in the modulated two-electron wave function (2′).

The consequences of invoking conservation of energy also appear in the two-electron problem. If the initial state of the black box and the final state of the whole system are energy eigenstates, then the two-electron amplitudes $A_2(j,k, B \rightarrow F;t)$ vanish except for pairs j, k which conserve energy. For each j, no more than one value of k contributes to the sums in (4′) and (6′). If the two-electron wave functions are uncorrelated in the sense that c_{jk} is a product $d_j d_k$, then $N_2 = N^2$, but the requirement of energy conservation reduces the limit in (7′) from N^2 to unity and there can be no enhancement. If the two-electron wave function is correlated in such a way that all terms in (2′) have the same energy, then $N_2 = N$ and the conservation of energy imposes no further restriction.

Equations (7) and (7′) establish the following two theorems:

Theorem 1: Modulated electrons can produce a final state with an enhanced probability that is greater by a large factor R than the probability of production by unmodulated electrons only if the wave function of the modulated state contains at least R approximately equally weighted plane-wave components.

This theorem applies equally to emission from one or from many electrons, but in the latter case R is the number of plane-wave components in the many-particle wave function.

Theorem 2: Enhancement of the production of a given final state by modulation of the electron wave function is impossible unless at least one of two conditions is met: Either the final state must be defined by a measurement that does not commute with the energy, or the initial multicomponent wave function must have definite energy.

The definite energy condition is impossible for a modulated initial state of a single electron. It could be met, for instance, by a correlated many-particle state. Such a state could satisfy the requirement of theorem 1 even if the individual electron wave functions had only two components.[9] It could also be met by a correlated state of two (or a few) electrons, each having many plane-wave components, resembling the Cooper pair state.

Theorem 2 has one mathematical exception which appears not to be physically important: The black box, which represents the initial state of the experimental apparatus, could be in a coherent state of indefinite energy.

We conclude that there are only two possible classes of theoretical explanations of the Schwarz-Hora experiment. Either the modulated electrons have a wave function with many side bands and the emitted light is coherent, or the light is emitted collectively by many electrons, from a state whose many-particle wave functions have definite energy but many plane-wave components. It appears that none of the one-electron or few-electron theories which have so far been advanced[2,3] incorporates either of the necessary features. The many-electron theory of Favro, Fradkin, and Kuo[4] does satisfy our conditions.[10]

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[1]H. Schwarz and H. Hora, Appl. Phys. Letters 15, 349 (1969); H. Schwarz, Ann. N.Y. Acad. Sci. (to be published).

[2]L. L. Van Zandt and J.W. Meyer, J. Appl. Phys. 41, 4470 (1970); A.D. Varshalovich and M.I. D'yakonov, Zh. Eksperim. i Teor. Fiz. Pis'ma v Redaktsiyu 11, 594 (1970) [Sov. Phys. JETP Letters 11, 411 (1970)]; A.R. Hutson, Appl. Phys. Letters 17, 343 (1970).

[3]L. L. Van Zandt, Appl. Phys. Letters 17, 345 (1970); D. Marcuse, J. Appl. Phys. 42, 2259 (1971); L.D. Favro, D.M. Fradkin, P.K. Kuo, and W.B. Rolnick, Nuovo Cimento Letters 4, 1147 (1970). Additional references are cited by these authors.

[4]L. D. Favro, D.M. Fradkin, and P.K. Kuo, Phys. Rev. D 3, 2934 (1971).

[5]Enhancement is then allowed only for experiments which measure some quantity which does not commute with the energy, such as the electric field at some point.

[6]In the intended application to experiment, plane-wave states are appropriate because the energy spread of the original electron beam is much less than the energy differences ΔE_k.

[7]L. D. Favro *et al.* (Ref. 3) first pointed out this consequence of energy conservation for the Schwarz-Hora experiment.

[8]This argument illustrates the pitfalls of the analogy between modulated Schrödinger waves and modulated radio waves. In a classical radio wave containing two components of frequencies ω_1 and ω_2, the beat frequency can be observed by passing the wave through a nonlinear detector and a filter sensitive only to the beat frequency. Such an apparatus will give an output signal only if both components are present. That kind of nonlinear apparatus does not exist for the nonclassical Schrödinger probability waves. Quantum mechanics is *linear*, and no apparatus can give a probability amplitude that is a nonlinear function of the incident probability amplitude.

[9]Such a superradiant state of many particles was introduced by R. H. Dicke [Phys. Rev. 93, 99 (1954)].

[10]C. Becchi and G. Morpurgo, Phys. Rev. D 4, 288 (1971), have recently proposed a many-electron theory which satisfies our conditions.