national accelerator laboratory

THY - 27
1971

Soft Pion Results for Inclusive Reactions

E. A. PASCHOS
National Accelerator Laboratory, Batavia, Illinois 60510

Operated by Universities Research Association Inc. Under Contract with the United States Atomic Energy Commission

ABSTRACT

Two of the main difficulties arising in the application of soft pion methods to inclusive reactions are discussed. It is shown, by explicit calculation, that for specific reactions the entire pole contribution can be calculated in terms of physically observable quantities. Available data on hadronic reactions suggests that the contribution of the pole terms is as important as the contribution of the equal time commutators. The other difficulty concerns the ranges of extrapolation in some variables which increase linearly with the center of mass energy.

Numerous papers have been published recently[1-5] attempting to apply soft pion techniques to inclusive reactions. There are, however, several serious difficulties that one encounters in such attempts:

(1) all the attempts involve extrapolations in one or more variables, whose range of extrapolation increases linearly with the center-of-mass energy.

(2) they involve contributions from the pole terms which, except for a few cases,[6] have been assumed to be negligible.

The point of this comment is to bring more attention to the difficulties encountered in the application of soft pion techniques to inclusive reactions. To this end we classify the contributions of the pole terms and show by explicit calculation that for some specific cases the entire pole contribution can be evaluated in terms of physically measurable quantities. Detailed evaluation of the pole terms indicates that the approximations of Ioffe depend critically on the explicit momentum distribution of baryons in the final state and _a priori_ they are not negligible. In evaluating the pole terms we distinguish three cases:

(1) those reactions where the equal-time-commutator vanishes and the pole contribution arises from insertions on baryons in the final state

$$\text{Example:} \quad \nu + p \rightarrow \mu^- + \pi^- \text{ (soft)} + \text{hadrons} \tag{1}$$

(2) those reactions in which the pole contributions arise from

insertions in both the initial and final states, but the equal time commutator term is again absent

$$\text{Example: } \bar{\nu} + p \to \mu^{+} + \pi^{-}(\text{soft}) + \text{hadrons} \qquad (2)$$

(3) those reactions where there are both equal-time-commutator and pole contributions

$$\text{Example: } e + p \to e + \pi^{-}(\text{soft}) + \text{hadrons}. \qquad (3)$$

In what follows we calculate explicitly the soft pion limit of the above reactions and then discuss the additional approximations that one must make in order to compare the results with experiment.

The hadronic part of the matrix element for reaction (1) is given by

$$\int d^4x\, e^{ik\cdot x}\, \theta(x_0)\langle p_n|\left[\frac{\partial A^+_\mu(x)}{\partial x_\mu}, J^+_\nu(0)\right]|p\rangle$$

$$= -ik^\mu \int d^4x\, e^{ik\cdot x}\, \theta(x_0)\langle p_n|\left[A^+_\mu(x), J^+_\nu(0)\right]|p\rangle \qquad (4)$$

where k_μ is the pion momentum, $J^+_\mu(x) = V^+_\mu(x) - A^+_\mu(x)$ is the weak current given in terms of the vector (V) and axial (A) currents. In the $k_\mu \to 0$ limit the pole term for the initial state is absent. The contribution from baryons in the final state simplifies considerably by appealing to the experimental observation[7] that in pp collisions

$$\frac{\sigma(pp \to 3\,\text{baryons} + X)}{\sigma(pp \to 1\,\text{baryon} + X)} \lesssim 10^{-3} \qquad (5)$$

where by baryons one means any combination of baryons and antibaryons allowed in the final state. We assume that a relation analogous to (5) also holds for electro- and neutrino-production processes. Within this assumption the soft pions can be emitted[8] only by neutrons in the final state. The square of the matrix element for reaction (1) is shown diagramatically in Figure 1:

$$|M_{fi}|^2 = \sum_X \langle p|\, J^+_\mu(0)|X\rangle \frac{\not p' + M}{2k\cdot p'} \not k \gamma_5 (\not p' - \not k + M) \not k \gamma_5 \frac{\not p' + M}{2k\cdot p'} \langle X|\, J^+_\nu(0)|p\rangle \qquad (6)$$

where the state X consists only of mesons and the rest of the notation is shown in Figure 1. Using the trivial identity

$$(\not{p}'+M)\not{k}\gamma_5(\not{p}'-\not{k}+M)\not{k}\gamma_5(\not{p}'+M) = (2k\cdot p')^2\left[1-\frac{M^2m_\pi^2}{(k\cdot p')^2}\right](\not{p}'+M) + O(k^3) \tag{7}$$

the pole contribution can be related to the reaction $\nu + p \rightarrow \mu^- + \text{neutron} + X$. Denoting, as usual, by Q^2 and ν the square of the mass and the laboratory energy of the weak current the final result is:

$$\frac{d\sigma}{dQ^2 d\nu\, dE_\pi}(\nu+p\rightarrow\mu^-+\pi^-+X) = \frac{|\vec{k}|}{(2\pi)^2 f_\pi^2}\left\{\frac{d\sigma}{dQ^2 d\nu}(\nu+p\rightarrow\mu^-+X') - \int d^3p'\,\frac{M^2m_\pi^2}{(k\cdot p')^2}\,\frac{d\sigma}{dQ^2 d\nu\, d^3p'}(\nu+p\rightarrow\mu^-+\text{neutron}+X)\right. \tag{8}$$

where the state X' contains a neutron plus any number of mesons, $F_\pi = .93m_\pi$, m_π and M are the pion and nucleon mass respectively.

There are several points to be discussed concerning the application of (8) to physical problems.

1. <u>Importance of pole terms.</u> The importance of the pole terms depends critically on the explicit momentum distribution of neutrons in the reaction $\nu + p \rightarrow \mu^- + n + \text{hadrons}$. The right hand side in equation (8) is zero, if all the neutrons are at rest in the laboratory frame and it is of order one if most of the neutrons are fast.

Looking again at hadronic reactions for some clues,[9] we note that the process

$$K^+ + p \rightarrow \Lambda^o + x$$

is closely related to the reaction under discussion, since it involves a baryon in the final state and exchange of quantum numbers in the cross-channel. We observe that in this reaction most of the Λ^o's are not at rest in the laboratory frame. Explicit calculation indicates that

$$\int \left(\frac{M}{E'}\right)^2 \frac{d\sigma}{dp'}\, dp' \Big/ \int \frac{d\sigma}{dp'}\, dp' \approx 1/4 \tag{9}$$

If the above process is indicative of corresponding hadron distributions in deep inelastic scattering, it suggests, contrary to Ioffe's approximations, that it is more reasonable to neglect the second term on the right hand side of equation (8) than to neglect the contribution of the pole terms completely.

2. Extrapolations. The soft pion result of equation (8) holds for the unphysical point $k^2 = k\cdot p = k\cdot q = 0$. As is always the case with soft pion theorems, any comparison with experiments involves an extrapolation in k^2, $k\cdot p$, $k\cdot q$. For pions at rest the variables $k\cdot p$ and $k\cdot q$ are given by $m_\pi p_0$ and $m_\pi q_0$, so that there are many different paths for extrapolating to the soft pion point. If we consider as soft pions those which are at rest in the laboratory frame[1] the extrapolation

in $k\cdot q$ increases linearly with energy and it becomes unreliable at high energies. The situation may be more favorable in the deep inelastic region,[2] where one can conjecture that the inclusive reaction

$$\nu + p \to \mu^- + \pi^- + X$$

scales, i.e., in the limit $q^2 \to -\infty$ with

$$\omega = \frac{2q\cdot p}{q^2}, \; x = \frac{k\cdot q}{q^2} \text{ and } y = \frac{k\cdot p}{q^2} \tag{10}$$

being held finite the cross section becomes, within known kinematic factors, a function of ω, x and y only. For pions at rest in the laboratory frame $y \to 0$, $x \to \frac{1}{2}\left(\frac{m_\pi}{M}\right)\omega$. Thus, if scaling sets in at a low value of Q^2 ($\approx M^2$), then the range of extrapolation in $k\cdot q$ is $\frac{1}{2} m_\pi M\omega$ which is comparable to the extrapolation in $k\cdot p$.

In such an extrapolation special attention must be given to the contribution of decaying resonances, like the baryon resonances and rho-mesons. A typical contribution from decaying resonances is proportional to

$$\frac{k\cdot p'}{\Delta M^2 + 2k\cdot p' - 2iM\Gamma} \tag{11}$$

where $\Delta M^2 \simeq .5(\text{BeV})^2$ and $\Gamma \simeq .1\text{BeV}$. The interference of this term with the pole-contribution seems to be important. Thus it will be preferable to compare soft-pion results with experiment in a kinematic region where resonance production is suppressed.

3. Multi-pion Emission. Since we do not observe what is contained in state X, there is the possibility that there are other soft pions present. There is also the possibility that several of them are attached to an external line as shown in Figures 2a and 2b. Such contributions are limited by phase space to be of $O(k^2)$ or higher and can therefore be neglected. This is easily seen by observing that the matrix element of the diagrams in Figures 2a and 2b are finite in the limit $k_\mu \to 0$ and in order to obtain a non-zero contribution in this limit, the phase space integrations must be limited in the region $\leq k_\mu$. A diagram with a soft π^- and N additional pions is of $O(k^{2N})$.

The analysis of reaction (2) is along similar lines. The amplitude now contains two pole terms. The new term arises from an insertion in the initial state, shown in Figure 3. In the notation of Figure 3 the amplitude is:

$$B_2 = S \frac{1}{\not{p} - \not{k} - M} \not{k} \gamma_5 u(p) \tag{12}$$

where S is the matrix element for the rest of the diagram. If we now consider as soft pions those pions which are at rest in the laboratory frame then:

$$B_2 \to \frac{m_\pi}{2M} S \gamma_5 u(p) \tag{13}$$

The most important contribution of this term will arise from its interference with the other pole term. This contribution is of $O\left(\frac{m_\pi}{M}\right)$ and could be neglected. We emphasize once again that this conclusion depends critically on choosing as soft pions those which are at rest in the laboratory frame.

Similarly, the cross section for reaction (3) contains: (a) a pole term like the one already discussed, (b) an equal-time commutator term[2] discussed by Ioffe, and (c) an interference term between the pole and the equal-time commutator. This term is shown in Figure 4 and is given by: $B_3 =$

$$\frac{1}{2k\cdot p'} \sum_x \langle p| A^+_\nu(0)|x\rangle \sum_r u_r(p')\overline{u_r(p')}\not{k}\gamma_5(\not{p}'+M)\langle x|V^Q_\mu(0)|p\rangle \tag{14}$$

The spinor operator $u_r(p')\overline{u_r(p')}\not{k}\gamma_5 u_s(p')\overline{u_s(p')}$ is non-diagonal in spin-space, so that any accurate determination of the interference term requires knowledge of the cross-section for the electro-production of a polarized nucleon plus any number of hadrons. Since such data is hard to come by, we do not proceed to calculate the interference term in detail.

There have also been attempts[3] to apply these techniques to hadronic reactions. In the "parallel configuration", for instance, equation (8) can be related formally by Adler's theorem[10] to inclusive reactions of pion on protons:

$$\frac{d\sigma}{dE_\pi}(\pi^+p\to\pi^-X) = \frac{|\vec{k}|}{(2\pi)^2 f_\pi^2}\left\{\sigma(\pi^+p\to X') - \int d^3p'\,\frac{M^2 m_\pi^2}{(k\cdot p')^2}\,\frac{d\sigma}{d^3p'}(\pi^+p\to p'+X)\right\} \tag{15}$$

As we have already discussed, at high energies the range of extrapolation of either $k\cdot p$ or $k\cdot q$ becomes very large so that any agreement with experiment is dubious.

To sum up, we have studied two of the main problems arising in the application of soft pion techniques to inclusive reactions: (a) the contributions of the pole terms, and (b) the validity of the extrapolations. Concerning the first problem, it has been pointed out that the entire contribution of the pole terms can be calculated for reaction (1) in terms of experimentally measurable quantities. This result together with similar calculations for reactions (2) and (3) indicate that the contribution of the pole terms depends critically on the momentum distribution of baryons in the final state. If the momentum distribution of baryons in hadronic reactions is indicative of corresponding distributions in electro- and neutrino-production processes, then the contributions of the pole terms are as large as the contributions of the equal-time commutators. Concerning the ranges of extrapolations, it was noted that when both of the initial particles are on the mass-shell the extrapolation in either $k\cdot p$ or $k\cdot q$ increases linearly with energy, so that at high energies soft pion techniques are inapplicable. In the deep inelastic region however, there is the possibility[1] that the scale is determined by $\sqrt{Q^2}$ and not by the mass of the nucleon. This possibility could make soft pion theorems useful in deep inelastic scattering.

ACKNOWLEDGEMENTS

The author acknowledges the collaboration of J. Iliopoulos in an earlier attempt to apply soft pion methods to inclusive reactions. He also thanks A. Pais, A. Sanda, J. Sullivan, S. Treiman, and A. Zee for helpful discussions.

REFERENCES

[1] A. Pais and S. B. Treiman, Phys. Rev. Letters 25, 975 (1970).

[2] B. L. Ioffe, Phys. Letters 37B, 101 (1971).

[3] F. Arbab, J. C. Callardo and L. Masperi, Nuovo Cim. Lett. 2, 1069 (1971).

[4] H. Goldberg, SLAC-PUB-805 (1970).

[5] S. Y. Lo, Nuovo Cim. Lett. 2, 1091 (1971).

[6] In the work of Pais and Treiman the process has been chosen appropriately so that there are no pole contributions. In Ref. 3, the pole contribution has been taken into account.

[7] C. W. Akerlof, et al., Phys. Rev. 3D, 645 (1971).

[8] We have omitted the discussion of pole terms arising from Σ's and Ξ's, since it is straight-forward to include such contributions.

[9] S. Stone, T. Ferbel, P. Slattery, and B. Werner, University of Rochester preprint (to be published in Phys. Rev.).

[10] S. L. Adler, Phys. Rev. 135, B963 (1964).

FIGURE CAPTIONS

Fig. 1 Pole term for reaction (1)

Fig. 2 Multi-pion emission

Fig. 3 Pole term for reaction (2)

Fig. 4 Interference term between pole and equal-time commutator for reaction (3)

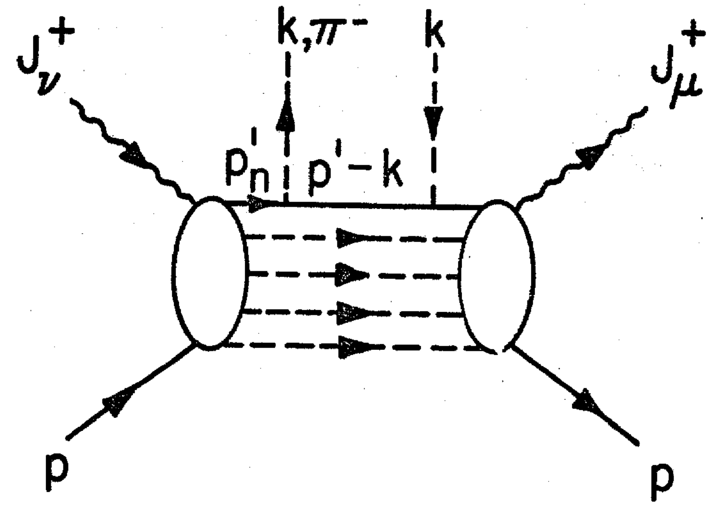

Figure 1

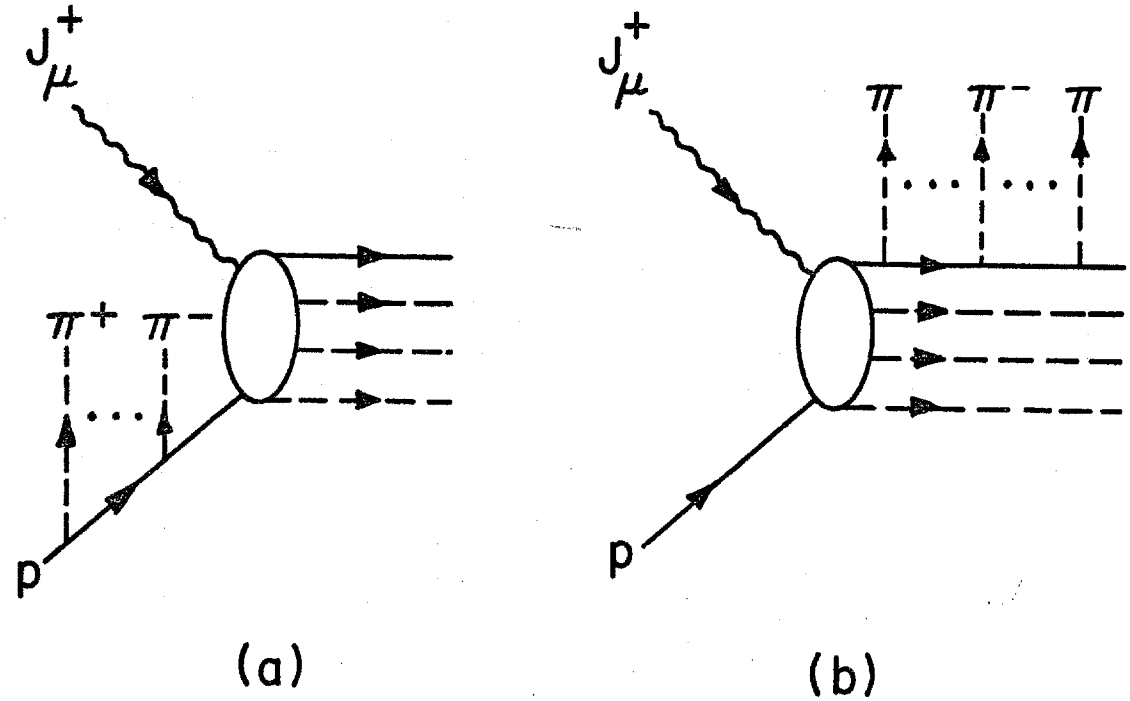

Figure 2

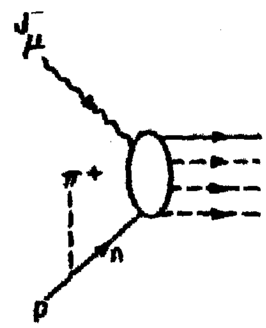

FIG. 3. Pole term for reaction (2).

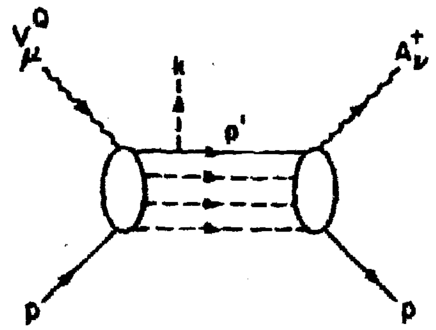

FIG. 4. Interference term between pole and equal-time commutator for reaction (3).