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Electromagnetic Contributions to Inelastic Hadron Processes

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ABSTRACT

Evidence obtained from inelastic electron scattering experiments supports the possibility that electromagnetic effects may become significant in hadron reactions, for production of particles with sufficiently high transverse momentum. Estimates of the electromagnetic contributions are given, along with much cruder estimates of the crossover transverse momentum where these effects may become competitive with the purely strong interaction contributions.

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A remarkable feature of strong interaction processes at high energies is that they become very weak in certain kinematic regions: e.g., for elastic scattering the differential cross section at fixed angle falls off rapidly with energy, perhaps exponentially; for multiparticle processes, the evidence suggests an exponential fall off with increasing transverse momentum of the produced particles. Thus for the inclusive reaction

$$p+p \rightarrow p+X, \tag{1}$$

BNL[1] data appears to be represented out to $q_\perp \approx 2$ Gev/c by the formula

$$\frac{d\sigma}{dq_\perp dq_\parallel} = 610\, q_\perp^{\,2} \exp(-6q_\perp)\ \mathrm{mb/(Gev/c)^2}. \tag{2}$$

It remains to be seen whether these trends will continue as more extreme kinematic domains come under experimental study. But already, the question can be raised: where differential cross sections become very tiny, do electromagnetic-and, eventually, weak interactions begin to make a significant contribution? The same mysterious effects which produce strong interaction damping may of course also suppress these latter contributions. But where damping is extreme, even a small mismatch in parameters (e.g., exponents) can greatly enhance the relative importance of the electromagnetic effects. The striking SLAC-MIT results on deep inelastic electron scattering[2] already suggest this possibility. In the present discussion we ignore the weak interactions, focusing on electromagnetic effects in hadron reactions.

For a start let us consider an elastic process, such as p-p scattering. One domain where electromagnetic contributions are certainly important is that of very small momentum transfer t. The dominant effect arises from one-photon exchange and the residue of the pole at $t = 0$ is of course fully known. For large momentum transfer, on the other hand, it is no longer obvious that the one photon exchange diagram represents the dominant electromagnetic effect. Nevertheless, let us focus on this contribution, which can be expressed in terms of the independently measurable electromagnetic form factors of the proton. In our example of identical particle scattering, we define t as the momentum transfer between incident proton and that outgoing proton which moves in the forward hemisphere in the center of mass frame. We consider the domain $s \gg q^2 \equiv -t \gg m^2$. Up to the largest momentum transfers so far studied experimentally, it appears that the form factors are monotonically decreasing functions of q^2 and that $G_M(q^2)/\ G_M(0) \approx G_E(q^2)$. Insofar as these properties hold for the domain under present consideration, the one photon exchange amplitude will be dominated by the magnetic form factor, and Pauli principle effects can moreover be ignored to leading approximation. To first order in the fine structure constant α the electromagnetic contribution to the differential cross section arises from interference between the strong and electromagnetic amplitudes. Nevertheless, in order to obtain a rough assessment of electromagnetic effects, let us consider the purely electromagnetic contribution (order α^2) to the differential cross section. If ever this becomes comparable to the experimentally observed cross section, we will be in a situation (at least on our model) where electromagnetic effects have become important. We find

$$\left.\frac{\partial\sigma}{\partial q^2}\right)^{em} \approx \frac{4\pi\alpha^2}{(q^2)^2} G_M^4(q^2) , \quad s >> q^2 >> m^2. \tag{3}$$

Up to the largest momentum transfers so far studied, the magnetic form factor is reasonably well approximated by the dipole formula

$$\frac{G_M(q^2)}{G_M(0)} \approx \frac{1}{\left[1+\left(\frac{q}{q_0}\right)^2\right]^2} , \tag{4}$$

where $q_0^2 \approx 0.71$ (Gev)^2. On our one photon exchange model, therefore, $(\partial\sigma/\partial q^2)^{em}$ falls off for large momentum transfer like $(q^2)^{-10}$! Although the experimental cross section at high energies also falls rapidly with q^2, for the regions so far studied it is always much larger[3] than the cross section of Eq. (3). (We are not concerned here with electromagnetic dominance near $q^2 = 0$.) According to certain theoretical speculations,[3,4] in fact, it is predicted that for very large energy s the purely <u>strong</u>

cross section will become independent of s and behave, for large q^2, like

$$\left.\frac{\partial\sigma}{\partial q^2}\right)^{str} \rightarrow \lambda \left.\frac{\partial\sigma}{\partial q^2}\right)^{str}_{q^2=0} \left[\frac{G_M(q^2)}{G_M(0)}\right]^4 \qquad (5)$$

when λ is a constant of order unity. If this situation actually obtains electromagnetic effects, at least those arising from one photon exchange, will of course remain negligible. On the other hand, on standard Regge pole models one expects that

$$\left.\frac{\partial\sigma}{\partial q^2}\right)^{str} \xrightarrow[s\to\infty]{} \left.\frac{\partial\sigma}{\partial q^2}\right)^{str}_{q^2=0} \left(\frac{s}{s_0}\right)^{-2[1-\alpha_P(q^2)]}, \qquad (6)$$

where α_P in the leading (Pomeranchuk) trajectory function and s_o is a scale factor. In this situation, clearly, electromagnetic effects would dominate for large enough energy s - well beyond presently studied energies, however, owing to the smallness of the cross section corresponding to Eq. (3).

Whatever may be revealed by experimentation in extreme domains of energy and momentum transfer (where all simple models are likely to fail), we may regard Eq. (3) as providing a rough gauge for the importance of electromagnetic effects. Just how rough a gauge cannot easily be judged. The simple one photon exchange model does not, for example, take into account interactions between the initial or the final protons. To see what modifications these particular effects induce, we shall employ the distorted wave Born approximation. For simplicity we neglect spins.

Let $f^{em}_{(0)}$ be the simple one photon exchange amplitude (Born approximation). In the inpact parameter representation this is expressed by

$$f^{(em)}_{(0)} = \int d^2b \, e^{i \underset{\sim}{\Delta} \cdot \underset{\sim}{b}} B(b, k), \tag{7}$$

where $k = (s/4 - m^2)^{1/2}$ is the center of mass wave number; and $\underset{\sim}{b}$ and $\underset{\sim}{\Delta}$ are 2-dimensional vectors, with $\Delta^2 = q^2$. For the strong interaction scattering amplitude we similarly write

$$f^{(str)} = \frac{-ik}{2\pi} \int d^2b \, e^{i \underset{\sim}{\Delta} \cdot \underset{\sim}{b}} [e^{2i\delta(b,k)} - 1] . \tag{8}$$

Finally, for the modified one photon exchange amplitude we adopt the approximation of replacing B by $Be^{2i\delta}$ in Eq. (7):

$$f^{(em)} = \int d^2b \; e^{i\Delta \cdot b} \; B(b,k) \; e^{2i\delta(b,k)} \quad . \tag{9}$$

Then, suppressing the k dependence of the amplitudes, we have

$$f^{(em)} = f^{(em)}_{(0)} + \frac{2\pi i}{k} \int \frac{d^2\Delta'}{(2\pi)^2} \; f^{(str)}(\Delta') \; f^{(em)}_{(0)}(\Delta - \Delta'). \tag{10}$$

Electromagnetic effects at large momentum transfer can become significant, on our model, only if $f^{(str)}$ falls off more rapidly with $\Delta^2 = q^2$ than does $f^{(em)}_{(0)}$. Let us therefore contemplate this situation, where, in Eq. (9), we can then make the approximation

$$f^{(em)}_{(0)} (\Delta - \Delta') \approx f^{(em)}_{(0)} (\Delta).$$

Suppose, further, that $f^{(str)}$ is purely imaginary and that

$$\left.\frac{\partial\sigma}{\partial q^2}\right)^{str} = \left.\frac{\partial\sigma}{\partial q^2}\right)^{str}_{q^2=0} e^{-bq^2} \; . \tag{11}$$

Then, via the optical theorem, we have

$$f^{(str)} = i \; \frac{k\,\sigma_T}{4\pi} \; e^{-\frac{b}{2} q^2} \; ; \tag{12}$$

and finally,

$$f^{(em)} = f_0^{(em)} \left\{ 1 - \frac{\sigma_T}{4\pi b} \right\} . \tag{13}$$

In these approximations the one-photon exchange amplitude remains unmodified in form but is somewhat reduced in magnitude.

It should be emphasized again that an estimate of strong corrections, Eq. (13), is intended, at best, for rough guidance. On the example of p-p scattering, we have found that electromagnetic effects are small, at least for presently studied domains of energy and momentum transfer. For a process such as $\pi^{\pm}p$ elastic scattering we are hampered by a lack of knowledge of the pion electromagnetic form factor. If this too obeys the dipole formula the effects will again be small in presently studied domains. Nevertheless, in view of all the uncertainties, the possibility remains that electromagnetic effects do in fact become important wherever the cross sections have become very tiny. In principle one could look for direct evidence of this, via tests of charge independence: e.g., comparison of $\pi^+ + d \rightarrow \pi^+ + d$ and $\pi^- + d \rightarrow \pi^- + d$.

In multiparticle reactions, as we shall now discuss, there is greater promise for important electromagnetic contributions. We want to consider inclusive processes of the sort

$$a+b \rightarrow X_a + X_b,$$

where the grouping of hadrons into X_a and X_b is supposed to be made unambiguous by the requirement that the invariant masses M_a and M_b of these systems, along with the invariant momentum transfer $-t = q^2 = (p_a - p_{X_a})^2 = (p_b - p_{X_b})^2 > 0$, are made to satisfy

$$s >> M_a^2,\ M_b^2,\ q^2. \qquad (14)$$

We shall now make the assumption that the electromagnetic amplitude arises from one photon exchange, as illustrated in Fig. 1. As in the elastic case, electromagnetic affects will first appear (order α) by way of interference between the strong and one photon exchange amplitudes. Nevertheless, to assess the importance of electromagnetic corrections we shall compute the purely electromagnetic contributions (order α^2) to the differential cross section, summing over all hadron states for fixed M_a^2, M_b^2, q^2. In particular let us take the incident particles a and b to be protons, so that our cross section can be explicitly worked out in terms of the independently measurable structure W_1 and W_2 which describe inelastic electron-proton collisions. In place of the masses M_a and M_b, let us introduce the variables ν_a and ν_b

defined by

$$M_i^2 = m^2 + 2m\nu_i - q^2. \qquad (15)$$

With

$$W_{1,2}(a) \equiv W_{1,2}(q^2, \nu_a)$$
$$W_{1,2}(b) \equiv W_{1,2}(q^2, \nu_b) \qquad (16)$$

we now find

$$\frac{\partial\sigma^{(em)}}{\partial\nu_a \partial\nu_b \partial q^2} = \frac{4\pi\alpha^2}{m^2} \cdot \frac{(1-4m^2/s)^{-1/2}}{(q^2)^2} \left\{ W_2(a)W_2(b)\left[1-\frac{2m^2\nu_a\nu_b}{sq^2}\right]^2 + 4\left(\frac{m^2}{s}\right)^2 \left[3\,W_1(a)W_1(b) - W_2(a)W_1(b)\left[1+\frac{\nu_a^2}{q^2}\right] - W_2(b)W_1(a)\left[1+\frac{\nu_b^2}{q^2}\right]\right]\right\}. \qquad (17)$$

With $s >> m^2$, $m\nu_a$, $m\nu_b$, q^2, this reduces to

$$\frac{\partial\sigma^{(em)}}{\partial\nu_a \partial\nu_b \partial q^2} \to \frac{4\pi\alpha^2}{m^2} \frac{1}{(q^2)^2} W_2(q^2, \nu_a)\, W_2(q^2, \nu_b). \qquad (18)$$

Finally, define

$$\omega = \frac{q^2}{2m\nu}, \quad \frac{\nu}{m} W_2 = F_2 . \tag{19}$$

Then, regarded as a function of ω_a, ω_b, q^2, the differential cross section can be written

$$\frac{\partial\sigma^{em}}{\partial\omega_a \partial\omega_b \partial q^2} \rightarrow \frac{4\pi\alpha^2}{(q^2)^2} \frac{F_2(\omega_a, q^2)\, F_2(\omega_b, q^2)}{\omega_a\, \omega_b} \tag{20}$$

We may recall here the experimental indications which suggest that F_2 scales, i.e., approaches a finite function of ω as q^2 becomes large - greater than a few $(\text{Gev}/c)^2$. In this scaling region, then, the cross section of Eq. (20) falls rather slowly with q^2, namely, like $(q^2)^{-2}$. We are restricting ourselves here to the region

$$s >> q^2,\ q^2\left(\frac{1-\omega_a}{\omega_a}\right),\ q^2\left(\frac{1-\omega_b}{\omega_b}\right), \tag{21}$$

$$q^2 > \text{ few } (\text{Gev}/c)^2.$$

From the point of view of currently fashionable strong interaction models, this is the "double diffraction region". For the strong contribution in this region Abarbanel, et al.[5] give the formula

$$\frac{\partial\sigma^{(str)}}{\partial M_a^2\,\partial M_b^2\,\partial q^2} = \frac{R(q^2)}{s_0^2\,s^2}\left[\frac{M_a^2 M_b^2}{s_0^{\,2}}\right]^{\alpha_P(0)}\left[\frac{s s_0}{M_a^2 M_b^2}\right]^{2\alpha_P(q^2)}, \qquad (22)$$

where s_0 is a scale factor, $\alpha_P(q^2)$ is the leading (Pomeranchuk) trajectory function, and $R(q^2)$ is a compound of residue functions. We could explicitly cast Eq. (22) into a form which expresses the differential cross section as a function of our variables q^2, s, ω_a, ω_b. For present purposes, however, it is enough to note that, for fixed q^2, ω_a, ω_b, the cross section falls with s according to

$$\frac{\partial\sigma^{str}}{\partial\omega_a\,\partial\omega_b\,\partial q^2} \xrightarrow[s\to\infty]{} f(q^2,\omega_a,\omega_b)\,s^{-2[1-\alpha_P(q^2)]} \qquad (23)$$

On this simple Regge pole picture, then, electromagnetic effects would become important for large enough energy s, just as in the elastic scattering case (compare Eqs. (3) and (6)). But in the present case the electromagnetic cross section of Eq. (20) is a rather slowly decreasing function of q^2; i.e., it is "anomalously" large when compared to the elastic scattering situation. If Eqs. (20 and (22) are reasonable guides, it may be that electromagnetic effects are important at presently accessible values of energy s and momentum transfer q^2. Of course, if the purely strong interaction model proposed in Ref. 4 is correct, then, as observed by Berman and Jacob [6], a corresponding model for multiparticle processes would lead to a strong interaction cross section like that of Eq. (20), with $\alpha^2/(q^2) \to \sim (1/m^2)^2$. In this situation electromagnetic effects would be insignificant.

Returning to more "conventional" expectations for the strong interaction contributions, we may ask at what point electromagnetic effects will begin to dominate. Unfortunately the parameters for Eq. (22) are not yet well enough known to provide a reliable estimate of the cross over point. We may, however, form a very crude judgement by considering the transverse momentum distribution for inclusive proton production, extracting this somehow from Eq. (20) and then comparing with the experimental results described by Eq. (2). Notice, in the first place, that in our kinematic region, Eq. (21), the momentum transfer is given approximately by $q^2 \approx (\Sigma\, q_\perp^i)^2$, where $q_\perp^{(i)}$ is associated with the ith produced particle. If we assume that the multiplicity of particles grows slowly with M_a^2, M_b^2, then Eq. (20) represents a power law with respect to the single particle transverse momentum spectrum, modified smoothly by some function depending on multiplicity. Given the rapid exponential fall off of Eq. (2), these modifications can be ignored in the first instance. Integrating over ω_a and ω_b, we find a cross over in the neighborhood of 4 Gev/c. Although this value of $q_\perp$ is fairly insensitive to the particle distributions within the clusters X_a and X_b, the incident energy at which our kinematic conditions can be met does depend strongly on these distributions.[7] Knowledge of the single particle distributions in deep inelastic electron scattering would be very helpful here.

We conclude with two final comments. First, an experimental decision whether electromagnetic effects are being seen clearly should not be based on the observation of a departure from some theoretical cross section formula of hadron physics. On the other hand, discovery of violations of isotopic spin invariance would give a clear signal that electromagnetic effects have become important. In this connection proton collisions on isoscalar targets (e.g., deuterium, carbon) are useful. Here, for the inclusive pion production cross sections one has the isotopic spin invariance test[8]

$$d\sigma_{\pi+} + d\sigma_{\pi-} = 2\, d\sigma_{\pi 0} \, . \tag{24}$$

Second, we believe that Eq. (20) may give a fair estimate of the (order α^2) electromagnetic contribution to the cross section and therefore a fair estimate of the experimental cross section wherever electromagnetic effects become large relative to the strong ones. That is, if the data anywhere were actually to fit well with Eq. (20), one would be inclined to suppose that the interpretation is that which we have associated with Eq. (20). Dreaming further, we can then envisage determination of the structure function W_2 for pions by experiments on the process $\pi + p \rightarrow X_a + X_b$.

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[6] S. M. Berman and M. Jacob, Phys. Rev. Letters 25, 1683 (1970).

[7] We thank Professor J. D. Bjorken for this remark. After the present work was completed, we learned that similar matters have been discussed by S. M. Berman, J. D. Bjorken, and J. Kogut "Inclusive Processes at High Transverse Momentum," SLAC preprint (1971). They emphasize the one particle inclusive distribution in transverse momentum and base their considerations on the parton model.

[8] We are indebted to Professor A. Pais for this comment.

FIGURE CAPTION

Figure 1: One photon exchange diagram for $a+b \rightarrow X_a + X_b$.

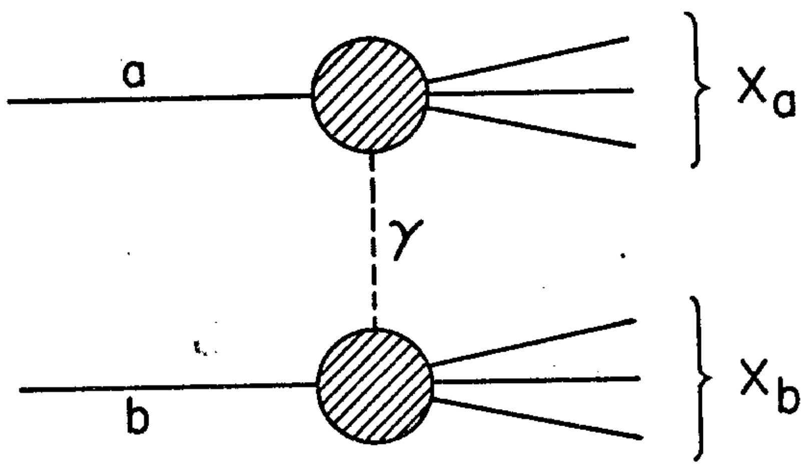

Figure 1

Electromagnetic Contributions to Inelastic Hadron Processes

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ERRATA

1. Eq. (17), page 10, should be replaced by:

$$\frac{\partial\sigma^{(em)}}{\partial\nu_a\partial\nu_b\partial q^2} = \frac{4\pi\alpha^2}{m^2}\,\frac{(1-4m^2/s)^{-1}}{(q^2)^2}\left\{W_2(a)W_2(b)\left[1-\frac{2m^2}{s}-\frac{2m^2\nu_a\nu_b}{sq^2}\right]^2 + 4\left(\frac{m^2}{s}\right)^2\left[3\,W_1(a)W_1(b) - W_2(a)W_1(b)\left[1+\frac{\nu_a^2}{q^2}\right] - W_2(b)W_1(a)\left[1+\frac{\nu_b^2}{q^2}\right]\right]\right\}. \qquad (17)$$

2. Footnote 7, page 16, should be replaced by:

[7] We thank Professor J. D. Bjorken for this remark. After the present work was completed, we learned that similar matters have been discussed by S. M. Berman, J. D. Bjorken, and J. Kogut, "Inclusive Processes at High Transverse Momentum," SLAC preprint (1971). They emphasize the one particle inclusive distribution in transverse momentum and base their considerations on the parton model. For a discussion of electromagnetic effects in non-diffractive strong

processes, see G. Berlad, A. Dar, G. Eilam, and J. Franklin, invited talk by A. Dar at Sixth Recontre de Moriond Meribal Les Allues, France, March, 1971.