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Finite-Energy Sum Rules for Inclusive Reactions

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ABSTRACT

A study of six-point function analyticity in the ϕ^3 theory is reported. With some understanding of the analyticity, a finite-energy sum rule for an inclusive reaction is derived. It relates the integral over the low-missing mass inclusive reaction cross section to the cross section in the triple-Regge limit. Various connections between the sum rule and the experiments are discussed.

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Let T be the scattering amplitude for the process shown in Fig. 1. Recently, generalized unitarity relation for T in the variable $(p_a + p_b - q)^2 = M^2$ in the region $M^2 < (p_1 + p_2)^2 = s$ was conjectured[1] and later varified.[2] Furthermore, use of 0(2, 1) expansion on T was made to obtain large M^2 behavior of the absorptive part of the amplitude.[1,3] The motivation for all this work was to study the inclusive reaction which is the next simplest to analyze, after the two-to-two reaction, theoretically if not experimentally. In the kinematical region under consideration, the inclusive cross section and the six-point function have the same relation as the total cross section and the four point function. In case of four-point function, the analyticity enables one to write a forward dispersion relation involving total cross sections. We then speculate that the techniques that were useful to the four-point function, for example, dispersion relations and finite-energy sum rule, can be applied also to the six-point function and thus yield information on experimentally measurable quantities simplest of which is the inclusive reaction cross sections.

In order to proceed toward the above mentioned goal, we must know the analyticity of the six-point function in the M^2 plane with s and t fixed. The obvious thing to do is to investigate a model such as the ϕ^3 theory. But this is still very complicated. We note, however, that to get some results which can be compared with experiments, we do not need the knowledge of analyticity on the entire M^2 plane. Knowledge of analyticity in the region $|s/M^2| >> 1$ is enough to give a very strong sum rule.

To see this we note that the cross section for the reaction in $a+b \to c+X$ can be written as (throughout this paper X corresponds to "anything")

$$\frac{d\sigma}{dt\,dM^2} = \frac{m_a m_b}{(2\pi)^2 s^2} A(s,t,M^2) \tag{1}$$

$$A(s,t,M^2) = \lim_{q^2 \to \mu^2} \frac{E_a E_b}{m_a m_b} \int d^4x\, e^{-iq\cdot x} (q^2-\mu^2)^2 \langle p_a p_b^{in} | \phi_c^+(x) \phi_c(0) | p_a p_b^{in} \rangle \tag{2}$$

Where m_a, m_b, μ are masses of particle a, b, and c respectively, ϕ_c is the field operator for the particle c. A is proportional to the absorptive part of T in the proper kinematical region. From Eq. (2) it is clear that A vanishes everywhere except $(p_1+p_2-q)^2 = M^2 \geq m_0^2$. m_0 is the lowest possible mass of the intermediate state. This singularity is shown in Fig. 2. The region $m_0^2 \leq M^2 \leq (\sqrt{s}-\mu)^2$ corresponds to the physicsl region for the inclusive reactions $a+b \to c+X$. $M^2 \geq (\sqrt{s}+\mu)^2$ corresponds to the physical region of the three-particle scattering when all the variables are analytically continued to the proper region. If the contour shown in Fig. 2 encloses no singularity, and if we know the M^2 dependence of T around the contour, it will yield some relation.[4] If we choose $M_0^2 >> m_0^2, s/M_0^2 >> 1$, the triple-Regge expansion tells us the M^2 dependence of T around the contour. This will result in an equivalent of the finite-energy sum rule of the four-point function. So we need only the analyticity in the region $|s/M^2| >> 1$ extending out to $|M^2| >> m_0^2$. But in this region, the analysis of the six-point function is greatly simplified. When $|s/M^2|$

is large, the leading contribution to the absorptive part of the six-point function is the Pomeron exchange shown in Fig. 3a. The importance of the Pomeron exchange was demonstrated experimentally in the reaction $p+p \rightarrow p+X$ by the fast decrease in Δ_{33} production and the consistency of the production cross section for $I=1/2$ baryons.[5] Then we can restrict ourselves to the six-point function of the class shown in Fig. 3b. We will state a theorem on the analyticity of the diagram shown in Fig. 3b.

Theorem:[6] Consider the diagram shown in Fig. 3b. (i) The four-point function associated with the lower black blob corresponds to the arbitrary sum of diagrams in the ϕ^3 theory such that it behaves as $[-(p_2+k)^2]^{\alpha(t)} \beta(k_1^2, (k+p_2-q)^2, t)$ in the limit of large $(p_2+k)^2$. Similarly for the upper black blob. (ii) The checked blob is a six-point function which represents an arbitrary Feynmann diagrams with n number of propagators and ℓ number of loops. Then in the limit of large s, the necessary condition for the diagram to contain additional singularity on the physical M^2 plane besides those required by the unitarity is that $\beta(m_1^2, m_2^2, t)$ contains a complex branch point on m_1^2 or m_2^2 plane or a branch point at m_1^2 or $m_2^2 < \mu_0^2$.

If we sum the leading log in the ladder diagrams, $\beta(m_1^2, m_2^2, t)$ is constant in m_1^2 and m_2^2. Thus the above theorem tells us that if the black blobs in Fig. 3b are replaced by the contribution from sum of leading logs of the ladder diagrams, then such a diagram has nothing other than the unitarity cut in M^2 plane. But we can not be sure whether non-leading log in the ladder diagram will not give $\beta(m_1^2, m_2^2, t)$ which has complex cut on the mass plane.

We now know the possible source of the additional singularities on the M^2 plane. We will take the point of view, however, that contribution from such a complex cut, if it existed, is weak. Let us now discuss the sum rule.[7] The triple-Regge expansion gives us the M^2 dependence of the amplitude around the circular contour of Fig. 2.

$$T \approx \frac{\pi}{4m_a m_b} \sum_{ijk} \eta_j \eta_k^* \left(\frac{s}{M^2}\right)^{\alpha_j(t)+\alpha_k(t)} (M^2)^{\alpha_i(0)} \frac{\beta_{bbi}(0)\, g_{ijk}(t)\, \xi_i}{\sin\pi(\alpha_i(0)-\alpha_j(t)-\alpha_k(t))} \tag{3}$$

where $\eta_j = \beta_{acj}(t)\, \dfrac{e^{-i\pi\alpha_j(t)} \pm 1}{\sin\pi\alpha_j(t)}$, $\xi_i = e^{-i\pi(\alpha_i(0)-\alpha_j(t)-\alpha_k(t))} \pm 1$,

β's are ordinary Regge-residue function and $g_{ijk}(t)$ is the triple-Regge residue function. Then the sum rule obtained by integrating around the contour shown in Fig. 2 is[8]

$$\int_{m_0^2}^{M_0^2} (M^2)^n \frac{d\sigma}{dt\, dM^2}(a+b\rightarrow c+X)\, dM^2$$
$$= \sum_{ijk} \frac{s^{-2}}{16\pi} \eta_j \eta_k^* \left(\frac{s}{M_0^2}\right)^{\alpha_j(t)+\alpha_k(t)} (M_0^2)^{\alpha_i(0)+n+1} \frac{\beta_{bbi}(0)\, g_{ijk}(t)}{\alpha_i(0)-\alpha_j(t)-\alpha_k(t)+n+1} \tag{4}$$

Similar sum rule can be obtained for the chsnnel $a+\bar{b}\rightarrow c+X$ if we subtract the two sum rules we obtain, for n = 0,

$$\int_{m_0^2}^{M_0^2} \left[\frac{d\sigma}{dt\,dM^2}(a+b\rightarrow c+X) - \frac{d\sigma}{dt\,dM^2}(a+\bar{b}\rightarrow c+X)\right]$$
$$= \sum_{ijk} \frac{s^{-2}}{8\pi} \eta_j \eta_k^* \left(\frac{s}{M_0^2}\right)^{\alpha_j(t)+\alpha_k(t)} (M_0^2)^{\alpha_i(0)+1} \frac{\beta_{bbi}(0)\, g_{ijk}(t)}{\alpha_i(0)-\alpha_j(t)-\alpha_k(t)+1} \tag{5}$$

where we sum over only the odd-signature Regge trajectory for i.

We now list the content of the sum rule. (a) Take $a+b \to a+X$ and $n=0$. If $\alpha_P(0) = 1$, then the denominator of the leading term $i=j=k=$ Pomeron vanishes as $t \to 0$. Since the left hand side is finite, we see that there must be a zero in $g_{PPP}(t)$ at $t = 0$. (b) Since the sum rule relates the resonance region to the Regge region, we might expect that the concept of duality in two-particle scattering may appear here in its generalized form. This will be true if the sum rule holds for unusually low M^2 with only the leading Regge trajectory in the sum over i. Since the generalized form of duality is widely accepted without any experimental basis, this is a good opportunity to check it. There is also a related question concerning how Pomeron and ordinary Regge contribution should be related to the contributions from the resonance and the background. If we take the analogy with the two-particle scattering, we associate the contribution of the resonance with the ordinary Regge contribution in i. It is clear, however, that the association of background and resonance productions with Pomeron and non-Pomeron Regge trajectories respectively cannot be made in Eq. (4), since the non-Pomeron Regge contribution in i will be monotonically decreasing function of M_0^2, whereas the resonance contribution to the left hand side is not monotonically decreasing. (c) For now we take large enough M_0^2 such that only the Pomeron contribution in i survives. Then [9]

$$g_{PPP}(t) = \frac{16\pi\left(1-\alpha_P(0)+2\alpha'_P(t)\,t\right)}{\sigma_a\sqrt{\sigma_b}} \int_{m_0^2}^{M_0^2} \frac{d\sigma}{dt\,dM^2}\left(a+b\to a+X\right)\, dM^2 \tag{6}$$

(d) Another immediate consequence of Eq. (6) which can be tested is that the right hand side of Eq. (6) is a universal number for all reactions, for example, $pp\to p+X$, $\pi^{\pm}He\to He+X$, $\pi^{\pm}p\to p+X$, $\pi^{\pm}p\to\pi^{\pm}+X$, $K^{\pm}p\to p+X$, $K^{\pm}p\to K^{\pm}+X$, $p\bar{p}\to p+X$, $p\bar{p}\to\bar{p}+X$, etc. (e) Eq. (5) can be rewritten

$$\int_{m_c^2}^{M_c^2}\left[\frac{d\sigma}{dt\,dM^2}(a+b\to c+X) - \frac{d\sigma}{dt\,dM^2}(a+\bar{b}\to c+X)\right] dM^2 \tag{7}$$

$$= \frac{s^{-2}}{8\pi}\,\beta_{bb\rho}(0)\,|\eta_\rho|^2\left(\frac{s}{M_c^2}\right)^{\alpha_P(t)+\alpha_\rho(t)+1}(M_c^2)^{\alpha_\rho(0)}\,\frac{g_{P\rho\rho}(t)}{\alpha_\rho(0)-\alpha_P(t)-\alpha_\rho(t)+1}$$

where we have assumed duality and kept only the leading term the ρ trajectory in i and k. This equation can be used to test duality.

Let us close by some comments on the design of future experiments on inclusive reaction in the light of the sum rule. It is clear that when inclusive reaction cross section is being measured in the region $|s/M^2|>>1$, some emphasis should be put on separating the background from resonance production. Furthermore, when $a+b\to a+X$ is being measured, $\bar{a}+b\to\bar{a}+X$ or $a+\bar{b}\to a+X$ should be measured simultaneously. This will enable us to test the concept of duality in the inclusive reaction. Once we have understood duality, we can go to evaluate the triple-Regge-Pomeron vertex using Eq. (6).

ACKNOWLEDGEMENTS

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REFERENCES

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[2] H.P. Stapp, Berkeley Preprint UCRL-20623 (1971).

[3] C.E. DeTar, et. al., Physical Review Letters 26, 675 (1971).

[4] Cuts associated with the physical region of crossed channels, (there are 25 channels), do not play a role here. For details see A.I. Sanda, NAL preprint THY-22, 1971.

[5] E.W. Anderson, et. al., Physical Review Letters 16, 855 (1966).

[6] Proof of the theorem is too long to be given in this letter. It is given in the reference sighted in footnote 4.

[7] While this work was in progress, the author has received M. Einhorn, Berkeley Preprint UCRL-20688 (1971), P. Olesen, CERN Preprint TH. 1376 (1971), both of which discusses finite-energy sum rule to some extent. H. Abarbanel has also derived a similar result. Private communication.

[8] For questions of left-hand cut, fix pole, etc. See the reference sighted in footnote 4. We assume that fix pole is absent in our process.

[9] Note that Eq. (6) is the only way $g_{PPP}(t)$ can be extracted from experiment. In triple-Regge region $\frac{d\sigma}{dt\,dM^2} \sim g_{PPP}(t)$ but due to the zero in $g_{PPP}(t)$ at $t=0$, any evaluation of $g_{PPP}(t)$ must be done at $t \neq 0$. But if $t \neq 0$, we do not know how much cuts contribute.

FIGURE CAPTIONS

Figure 1: Scattering amplitude T.

Figure 2: M^2 plane analyticity from unitarity, s and t fixed. A sum rule can be derived by integrating around the contour shown.

Figure 3a: Dominating process when $|s/M^2| >> 1$.

3b: Set of diagrams studied in the ϕ^3 Theory.

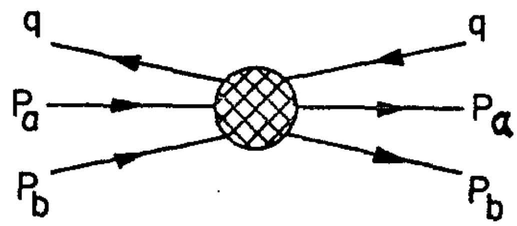


Figure 1

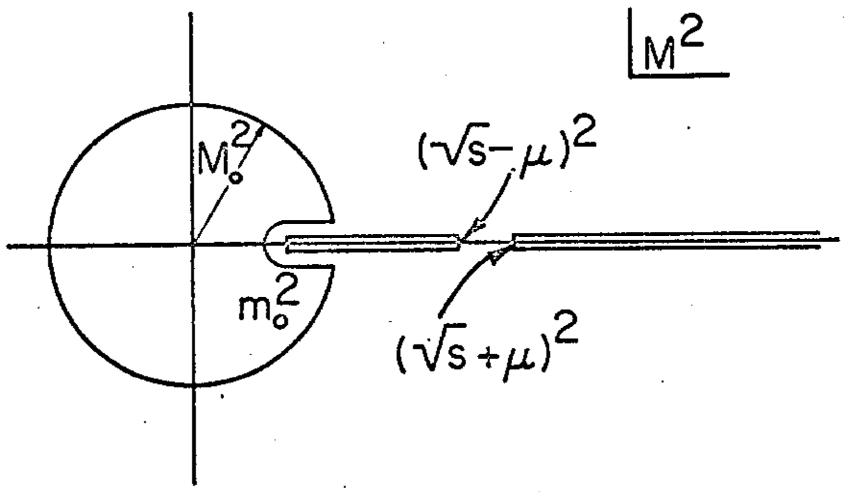
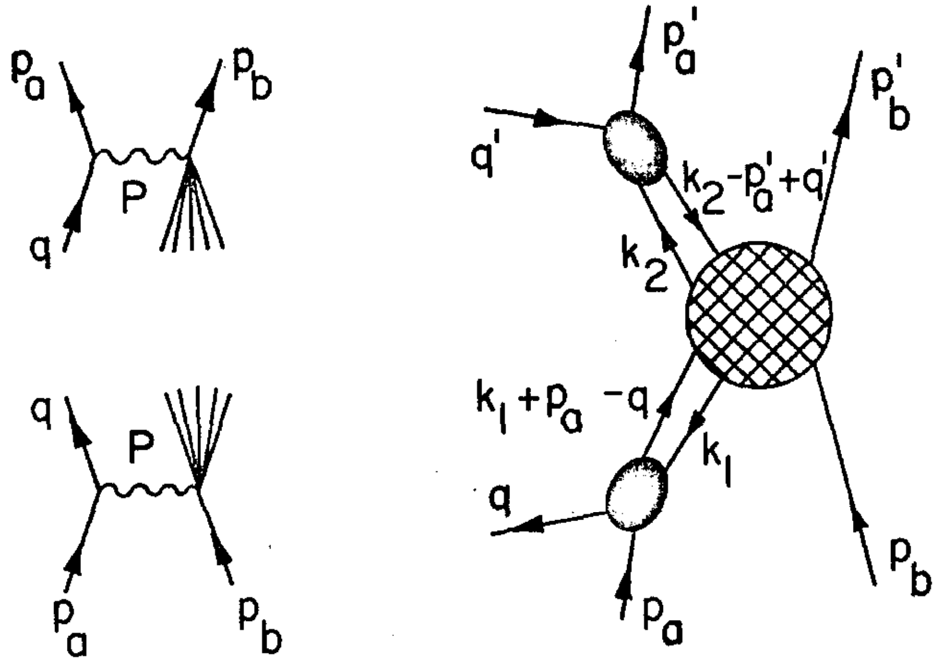


Figure 2



(a) (b)

Figure 3