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"Nuclear Physics" of Nucleons†‡
(Resonance Model of Inelastic Electron and Neutrino Scattering)

G. Domokos and S. Kovesi-Domokos
National Accelerator Laboratory, Batavia, Illinois 60510

and

Department of Physics, Johns Hopkins University
Baltimore, Maryland, 21218*

and

E. Schonberg
Department of Physics, Johns Hopkins University
Baltimore, Maryland, 21218**

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* Permanent Address

** Address after July, 1971: Belfer Graduate School of Science, Yeshiva University, New York, New York.

ABSTRACT

We discuss the physical aspects of a resonance dominance model of inelastic electron and neutrino scattering and summarize its main predictions. We stress analogies with the electroexcitation of nuclei and discuss possible relations with the quark-parton models. We investigate the possibility of a breakdown of scaling as a consequence of changes in the form of the baryon spectrum and suggest a way of testing it experimentally.

1. Introduction

Some time ago we suggested that probably there is a very substantial direct channel resonance contribution to "deep inelastic" electron and nucleon scattering.[1] The precise formulation of this hypothesis and its physical consequences have been worked out in a series of papers.[2] The purpose of this note is to discuss the physical basis of the resonance-dominance hypothesis and to summarize the main consequences of the resonance model. In the framework of this model, one obtains a fairly consistent qualitative picture of semileptonic processes, somewhat resembling the physical picture one forms about the mechanism of nuclear reactions in the compound nucleus region. Roughly speaking, the main assumption of "compound" theories is that the dynamics of the system excited is so complicated that any feasible experiment is unable to yield a detailed information about-say-the precise level structure. However, precisely the assumption that the system is extremely complicated ("Assumption of a Big Mess", or ABM for short) leads to the idea of defining suitable averaged quantities, about which one can make simple hypotheses and - more important - which are directly measurable.

Our model operates in terms of such averaged quantities, the basic properties of which are largely independent of the details of the dynamics. (Needless to say, a precise mathematical formulation of a statistical theory is a rather difficult task; in order to extract useful predictions, one quite often has to resort to intuitive considerations, largely based upon analogies with nonrelativistic systems, etc.)

In accordance with the purpose of this paper, we wish to stress the physical aspects of the problem, skipping most of the technical details. (The interested reader can find them in Ref. 2 just quoted.) We use conventiontional notation for the kinematical quantities involved; m will always denote the mass of the nucleon, and M that of a resonance.

2. The Physical basis of the Resonance Model.

Experiments on both inelastic electron and neutrino scattering indicate a few striking features of these processes. For our purposes we summarize them as follows.

i) If both the energy - and momentum transfers are "large", the excitation functions - invariant amplitudes - depend essentially on the quantity $x = 9^2/2m\nu$ only. ("Bjorken scaling").

ii) The excitation function shows well distinguishable "bumps", corresponding to the excitation of nucleon resonances With increasing momentum transfer, the bumps become less and less prominent. Even at fairly low values of q^2, however, the excitation function follows on the average the curve obtained in the scaling limit. ("Bloom-Gilman duality"[3]).

iii) There seems to be a substantial non-diffractive contribution to the scattering; in particular, the excitation functions of the proton and neutron appear to be significantly different from each other. One gets the impression that as $x \to 0$, the non-diffractive part decreases rather rapidly and diffraction takes over.

These qualitative features (in particular, the one described in ii) are strikingly similar to the picture one obtains in electroexcitation of nuclei. In order to illustrate this feature, in Fig. 1, we plotted the electroexcitation amplitude, $(E-E') W_2$ of a $_6C^{12}$ nucleus[4] against the variable $\omega = (E - E')(4EE' \sin^2 \Theta/2)^{-1}$, where E and E′ are the energies of the electron before and after the scattering. Apart from the energy-scale, the qualitative similarity of this curve to the ones found in the Stanford experiments is quite evident.

Now, when we talk about a nucleus, nobody has any serious doubt that a substantial part of the continuum excitation comes from the tails of the - increasingly overlapping - resonances. (Thus, strictly speaking, one is really dealing with a quasicontinuum, which is largely composed of ever-broadening, "discrete" levels.)

Is it possible that a non-negligible part of the electron-and neutrino-excitation of nucleons has the same origin? If so, how can we explain scaling and other features of the excitation functions? It is immediately clear that in order to have a "continuum-looking" excitation built up of resonances, it is necessary that we have many levels, which become more and more overlapping with increasing excitation energy. This

This indeed seems to be the case, as we illustrate it in Fig. 2. Besides the well-known, roughly equidistant spacing of levels, another empirical rule emerges:[5] The total widths of nucleon resonances on the average grow as the mass of the resonance. One indeed obtains a good average fit with the function:

$$m\Gamma(M) = \Gamma_0 M^{-1} m\,(M^2 - m^2)$$
$$(\Gamma_0 \approx 0.13)$$

where M stands for the mass of the resonance. Therefore above M ≈ 3 BeV, the resonances become for all practical purposes completely overlapping. Further, it is quite plausible that the number of states in a narrow band of energy must be rapidly increasing with the energy. This happens in every complicated dynamical system (e.g., a nucleus) and is strongly suggested by the success of dual models that it is also true for hadrons. How about the transition form factors?

A transition matrix element is effectively proportional to the overlap function of the initial and final states. For a highly excited state, the final state wave function is expected to oscillate rapidly and more-or-less randomly in coordinate space at distances larger than the inverse of the excitation energy. Thus, if the excitation energy is $M - m \approx M$ $(M \gg m)$, one expects a significant contribution to the overlap integral in a coordinate range up to $R \sim 1/M$.

The transition form factors are thus expected to depend on the momentum transfer in the form $q^2R^2 \sim q^2/M^2$ for high enough excitations. It should be mentioned that the same behaviour of the transition matrix elements was conjectured by Elitzur[6] on the basis of his investigations into the resonance saturation of the Bloom-Gilman finite energy sum rules.[3]

If we now select out a single state from the quasicontinuum of resonances (which according to ABM is <u>not</u> observable) its contribution to the structure functions-say W_2 will consist of terms which are typically of the form:

$$\nu W_2^{(\alpha)} \approx q^2 \left(\frac{k^2}{2}\right)^{\ell} \frac{(2\ell+2)!!}{(2\ell+1)!!} \left| G_\alpha \right|^2 \times \frac{\Gamma_\alpha}{(M^2-s)^2 + M^2\Gamma^2} \tag{2.1}$$

Here $s = -(p+q)^2 = m^2 + 2m\nu - q^2$, k is the magnitude of the relative momentum in the rest frame of the resonance. All the factors in this expression have an evident meaning. The first two factors give just the familiar barrier penetration factor for an orbital state ℓ, $G\alpha$ is a transition form factor to a resonance <u>fully</u> specified by the collection of quantum numbers α, and, finally, the last factor is the Breit-Wigner factor taking into account the finite spread of the level in question. It should be emphasized that $\Gamma\alpha$ is the reduced partial width of the resonance into the γN channel, whereas Γ is its total width.

The photon being virtual, $\Gamma\alpha$ may depend on q^2, of course. What do we observe? Clearly, not each individual term like (2.1), but the sum of such terms over the levels which are contained in the energy band of width $\sim \Gamma$ centered around M. Formally, we can write this as

$$\nu W_2 \approx q^2 \sum_\alpha \left(\frac{k^2}{2}\right)^\ell \frac{(2\ell+2)!!}{(2\ell+1)!!} |G_\alpha|^2 \frac{\Gamma_\alpha}{(M^2-s)^2+M^2\Gamma^2}$$

where the summation is extended over every quantum number (angular momentum, parity, etc.) except the energy. Persuing the analogy with nuclear physics further, we rewrite the last expression as

$$\nu W_2 \approx |G|^2 q^2 \frac{\Gamma}{(M^2-s)^2+M^2\Gamma^2} F \tag{2.2}$$

where

$$F = \sum_\alpha \left(\frac{k^2}{2}\right)^\ell \frac{(2\ell+2)!!}{(2\ell+1)!!} \frac{\Gamma_\alpha}{\Gamma} .$$

is just what is usually called a strength function at a given energy. The quantity G is now an average form factor, not depending anymore on α, but only on M and q^2, in the form as we conjectured above.

The strength function in turn measures the relative probability with which the "average resonance" decays into γN.

Thus, in the spirit of ABM, it is G and F, the average quantitiès, which are observable and about which one should make some reasonable physical assumptions.

It turns out that even the crudest assumptions lead to very reasonable results.

We assume that the average form factor has essentially the same shape as the elastic form factor, viz.

$$G \approx \frac{g_0}{\left(1 + r_0^2 \frac{q^2}{M^2}\right)^2} \tag{2.3}$$

where g_o is a low energy parameter, which can be determined from "static" experiments (e.g., by measuring total charges, magnetic dipole transitions rates in photoproduction of the lowest-lying resonances near threshold, etc.) The parameter r_0 is determined from elastic electron scattering (for vector form factors) and e.g., from pion photoproduction and PCAC (for axial form factors).

As far as the strength function, F is concerned, we make the simplest possible assumption about it, namely that no channel is preferred over any other one in the decay of an "average resonance" of sufficiently high mass.

Since we have summed over all quantum numbers α (including angular momentum), the decay channels are now characterized only by the number of particles contained in them.

The latter grows roughly as the mass of the resonance. (Imagine, e.g., that there are only pions in the world; then a resonance of mass M can decay into $N = [\frac{M-m}{\mu}] \sim \frac{M}{\mu}$ pions, where μ is the pion mass.) We thus assume that

$$F \sim \frac{f}{M}$$

where f is a constant to be fitted to the experimental data.

The important fact to be emphasized is that - at least if ABM is valid - f is a constant which should be the same no matter what transition we consider. Thus we obtain a one parameter fit to all the electron- and neutrino scattering data!

Finally, we sum over the masses of the resonances. Here the nature of the spectrum becomes important. Indeed, with an equidistant level spacing $M^2 \sim m^2 n$; $n = 1, 2, \ldots j$ in the masses and the empirical rule for the total widths, we get:

$$\nu W_2 \approx \sum_n \frac{\Gamma_0 f q^2}{(m^2 n - s)^2 + \Gamma_0^2 m^4 n^2} \left[G\left(\frac{q^2}{m^2 n}\right)\right]^2 \qquad (2.4)$$

One then writes $s = q^2(\omega' - 1) \equiv q^2\left(\frac{1}{x} - 1 + \frac{m^2}{q^2}\right)$, where ω' is a "scale variable", and realizes that for large momentum transfers $\sum_n$ can be replaced by $\int d\left(\frac{nm^2}{q^2}\right)$. This establishes the scaling property of the structure functions. Indeed, at the elementary level we are treating the problem here, one finds νW_2 to be roughly proportional to

$$(\omega'-1)^{-1} \left[G\left(\frac{1}{\omega'-1}\right)\right]^2$$

which-with a dipole form for G- reproduces the data quite well.

Of course, a realistic calculation is rather more complicated than this. There are several form factors, kinematic singularities have to be extracted, isospin taken into account, etc. The details of these calculations have been described in Ref. 2. and we don't want to repeat them here, but merely summarize the main results obtained.

3. Summary of Results.

The results obtained from the resonance model depend on the parameter f in the strength function. Once that (and, of course, the low energy parameters) are fixed, the model makes absolute predictions.

i.) Electron and neutrino scattering, unpolarized targets.

The structure functions satisfy $\nu W_2(\omega') = \frac{2}{\omega'} W_1(\omega')$ in the scaling limit, (i.e., the photoabsorption cross section is purely transverse and the scalar cross section in neutrino reactions vanishes.) The longitudinal scalar cross sections decrease relative to the transverse ones as $q^2 \nu^{-2}$. Thus effectively we have one structure function in electron scattering. The scaling limit is found to be approached rather rapidly. Above about $q^2 = 1.5\ \mathrm{GeV}^2$ the resonance bumps are effectively smeared out and one obtains a smooth curve for the structure functions.

The structure function νW_2 in electron scattering is shown in Fig. 3 for a proton target together with some experimental points taken from Ref. 7. For comparison we also plotted the same structure function for neutrino scattering. The quantity measured in present neutrino experiments is the event rate on a target containing an equal number of protons and neutrons. One prediction for this quantity together with data from CERN[8] is shown in Fig. 4.

The difference between the structure functions of the proton and neutron receives contributions from I = 1/2 resonances only.

Its calculated values are given by the curve in Fig. 5, together with the Stanford data of last year.[7] The calculated curve tends to fall below the data; due to the large errors and possible complications resulting from the structure of the deuteron, no definite conclusions can be drawn, however.

ii.) Spin dependence and vector-axial interferences.

The model predicts a substantial spin dependence of one cross sections and a rather large VA interference term. The calculated theoretical asymmetry:

$$A = \frac{d\sigma_{\uparrow\downarrow} - d\sigma_{\uparrow\uparrow}}{d\sigma_{\uparrow\downarrow} + d\sigma_{\uparrow\uparrow}}$$

for a polarized beam-polarized target electron scattering experiment is plotted in the scaling limit in Fig. 6, whereas in Fig. 7, we plotted the ratio $(-xW_3)/W_2$ for neutrino scattering on a target containing an equal number of protons and neutrons.

No experimental data exist at present for these quantities. The integral of νW_3 over the scale variable has been estimated by Myatt and Perkins[8] from the CERN data. Our prediction agrees with their result within the (very large) error. The measurement of these quantities is very important from the theoretical point of view. Almost any model reproduces νW_2, but various models differ widely in their predictions concerning the spin dependence and VA interference.

iii) Individual Resonance Production.

Although the model has been devised so that it works best when the experimental data are averages over many resonances, in principle one can calculate production cross sections of individual resonances as well. In Fig. 8, we plotted the predicted electroproduction cross sections of $\Delta(1236)$ and $N^*(1525)$, respectively, together with data taken from Ref. 9. (These predictions test the correctness of our assumptions about the form factors.) There is a very reasonable agreement between the theoretical curve and the data. To summarize, we find that at not too high values of the scale variable there is very good agreement between the theory and the experimental data where such exist. As one moves far away from the threshold, say below $x \approx 0.2$, the agreement becomes somewhat worse. Apart from the trivial explanation that in that region the experimental data are taken at low values of q^2, one can reasonably conjecture that very far away from threshold diffractive contributions dominate the cross sections which are presumably not represented by a simple sum over resonances. Therefore the validity of a resonance model can be really well tested on those combinations of the amplitudes which do not contain diffractive contributions.

4. Can Scale Invariance Break Down?

We wish to discuss briefly the theoretical possibility of a breakdown of scale invariance due to changes in the spectrum. Naturally, there are several reasons why scale invariance may break down; the one to be discussed here is the simplest. Our assumption about the strength function seems to be a fairly general one and essentially amounts to ABM. For the sake of definiteness we also keep the assumption that the total width grows linearly with the mass. (These two assumptions together imply that the quantity:

$$\sum_{\alpha} \left(\frac{k^2}{2} \right)^{\ell} \frac{(2\ell+2)!!}{(2\ell+1)!!} \Gamma_{\alpha}$$

is -approximately - constant.) Further, the assumption that the average form factor depends on q^2/M^2, is again ABM, so we do not want to abandon it. We do not assume, however, any longer an equidistant spacing of levels. Let us return to Eq. (2.4) and rewrite it in a form suitable for the present discussion. We shall use the approximation:

$$\frac{\Gamma_0}{x^2 + \Gamma_0^2} \approx \pi \, \delta(x) \qquad (4.1)$$

Assuming that the mass of the n^{th} band of resonances is now given by a general function:

$$M_n^2 = h(n)$$

with the inverse:

$$n = H(M^2)$$

we find:

$$\nu W_2 \approx \sum_n \frac{f q^2 \Gamma_0}{(M_n^2 - s)^2 + \Gamma_0^2 M_n^2} \left[G\left(\frac{q^2}{M_n^2}\right)\right]^2$$

$$\approx \int du \; \frac{f \Gamma_0 H'(q^2 u)}{(u - \omega' + 1)^2 + \Gamma_0^2 u^2} \left[G\left(\frac{1}{u}\right)\right]^2$$

where

$$u = \frac{M_n^2}{q^2}$$

Using (3.1) this becomes:

$$\nu W_2 \approx \pi f \, H'(q^2(\omega' - 1)) \, \frac{\left[G\left(\frac{1}{\omega' - 1}\right)\right]^2}{\omega' - 1} \qquad (4.2)$$

For the case of a strictly linear spectrum, $M_n^2 \approx nm^2$, the derivative H' is a constant, $H' \approx m^{-2}$, and we get back the old result: $\nu W_2(\nu, q^2) \to F_2(\omega')$.

The new result, (4.2) can be written in the form:

$$\nu W_2 \approx H'(q^2(\omega'-1))\, F_2(\omega')$$

Thus, we learn the important fact that as long as the quantity $q^2(\omega'-1)$ does not vary over a wide range, deviations from scaling are small for almost any reasonable form of the spectrum. It would be interesting to test the validity of scaling in the future generation of electron and neutrino scattering experiments by keeping ω' constant (say, around $\omega' \approx 5$, where scaling is well established by the Stanford experiments) and varying q^2 over a wide range above $q^2 \approx 2$.

It is amusing to observe that if our qualitative picture of the spectrum is true (increasing level density and strong overlap, so that practically no sharp resonance peaks are seen at high energies), the breakdown of scale invariance is about the only way of observing eventual changes in the baryon spectrum.

5. Discussion.

We based our model on apparent analogies between the excitation of nucleons with those of nuclei. The qualitative feature which makes such an analogy meaningful is evidently the presence of strong, short-range correlations in a complicated system.

Whereas we do not want to suggest that a nucleon *is* literally some kind of a nucleus (composed e.g., of quarks in the ordinary sense of the word), the analogy is well worth keeping in mind. In a sense, the present picture is an "anti-parton-model". The parton model assumes that the scattering takes place on a bunch of free particles (quarks?) and eventual correlations enter only through the distribution function of the quarks. The picture presented in this paper emphasizes what in nuclear physics would correspond to the collective aspects of the system. Both pictures may not be incompatible. In particular, both the parton- and resonance models predict scaling, they lead to the vanishing of the longitudinal photoabsorption cross section, etc. There are also differences at present. Sum rules derived on the basis of various versions of the free-quark model (like the Gross-Llewellyn Smith sum rule) are - in general - not satisfied by our structure functions. (Where such comparison is possible, present data seem to favor the predictions of the resonance model over those of the free quark model.) An optimistic point of view would be to assume that the parton-and resonance models stress two extreme aspects of the same physical phenomenon and both will be incorporated one day into some unified pictures.

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FOOTNOTES

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[2]G. Domokos, S. Kovesi-Domokos and E. Schonberg, Phys. Rev. D3, 1184, (1971), ibid. D3, 1191 (1971).
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[9]W. Panofsky in Proc. 14th International Conference on High Energy Physics, (Ed. J. Prentki and J. Steinberger) CERN, 1968.

FIGURE CAPTIONS

Figure 1. Structure function W_2 of the nucleus ${}_6C^{12}$. The dashed line is a curve drawn through the experimental points. The solid line is a smoothed semilocal average of the data.

Figure 2. Empirical properties of the nucleon spectrum. Dots: I = 1/2 states. Triangles: I = 3/2 states. Data from A. Barbaro-Galtieri et al. Rev. Mod. Phys. 42, 87 (1970).

Figure 3. Electroproduction structure function νW_2 for a proton target as predicted by the resonance model. For comparison the same structure function as predicted for neutrino - induced reactions on a target containing an equal number of protons and neutrons is also drawn. Data taken from Ref. 7.

Figure 4. Event rate in neutrino induced reactions for equal number of protons and neutrons in the target. Data from Ref. 8.

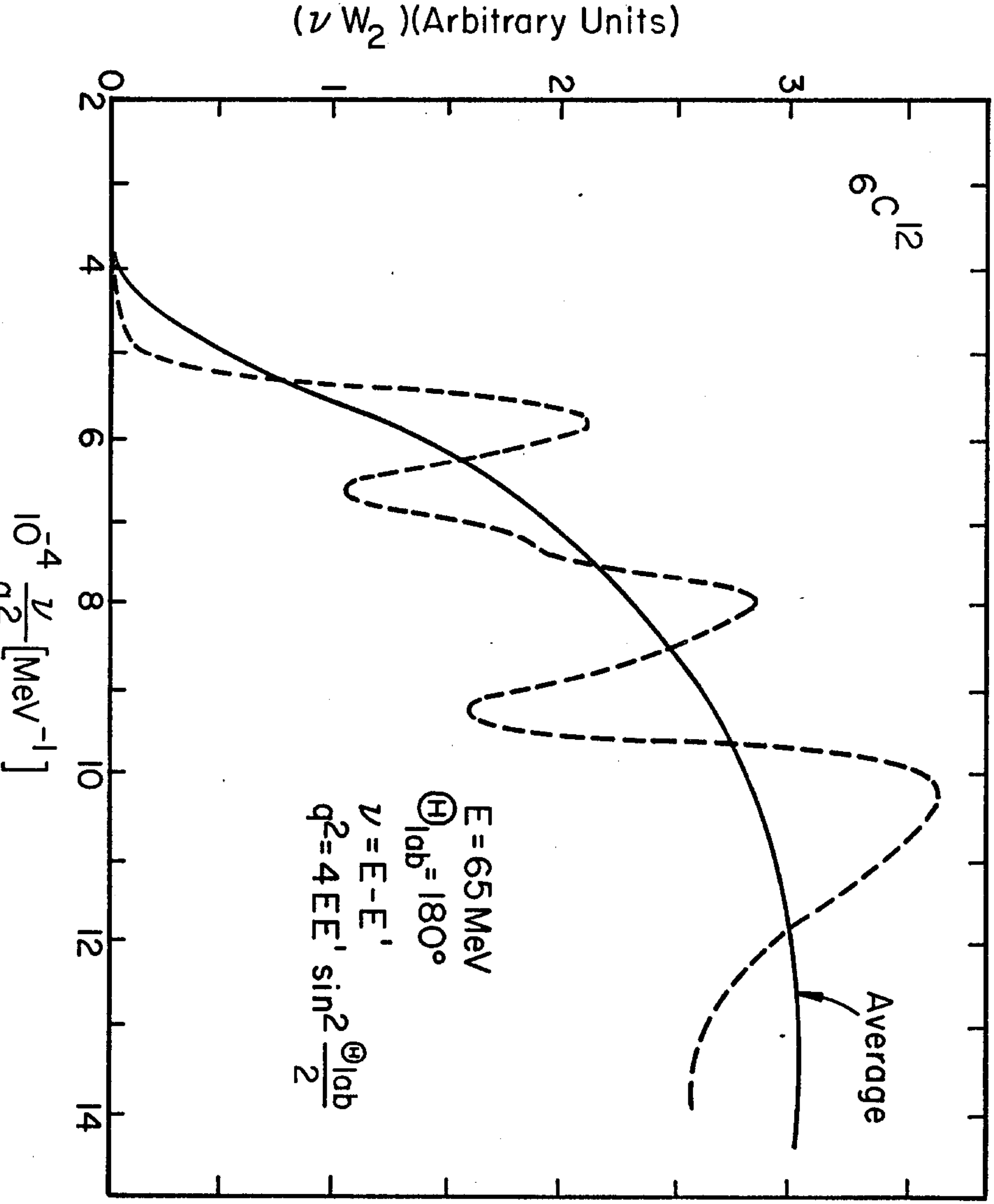
Figure 5. Predicted pn difference in electroproduction. Data from Ref. 7.

Figure 6. Predicted polarization asymmetry in eletroproduction on polarized protons.

Figure 7. Ratio of VA interference term to νW_2 in neutrino scattering. (Equal number of protons and neutrons in the target)

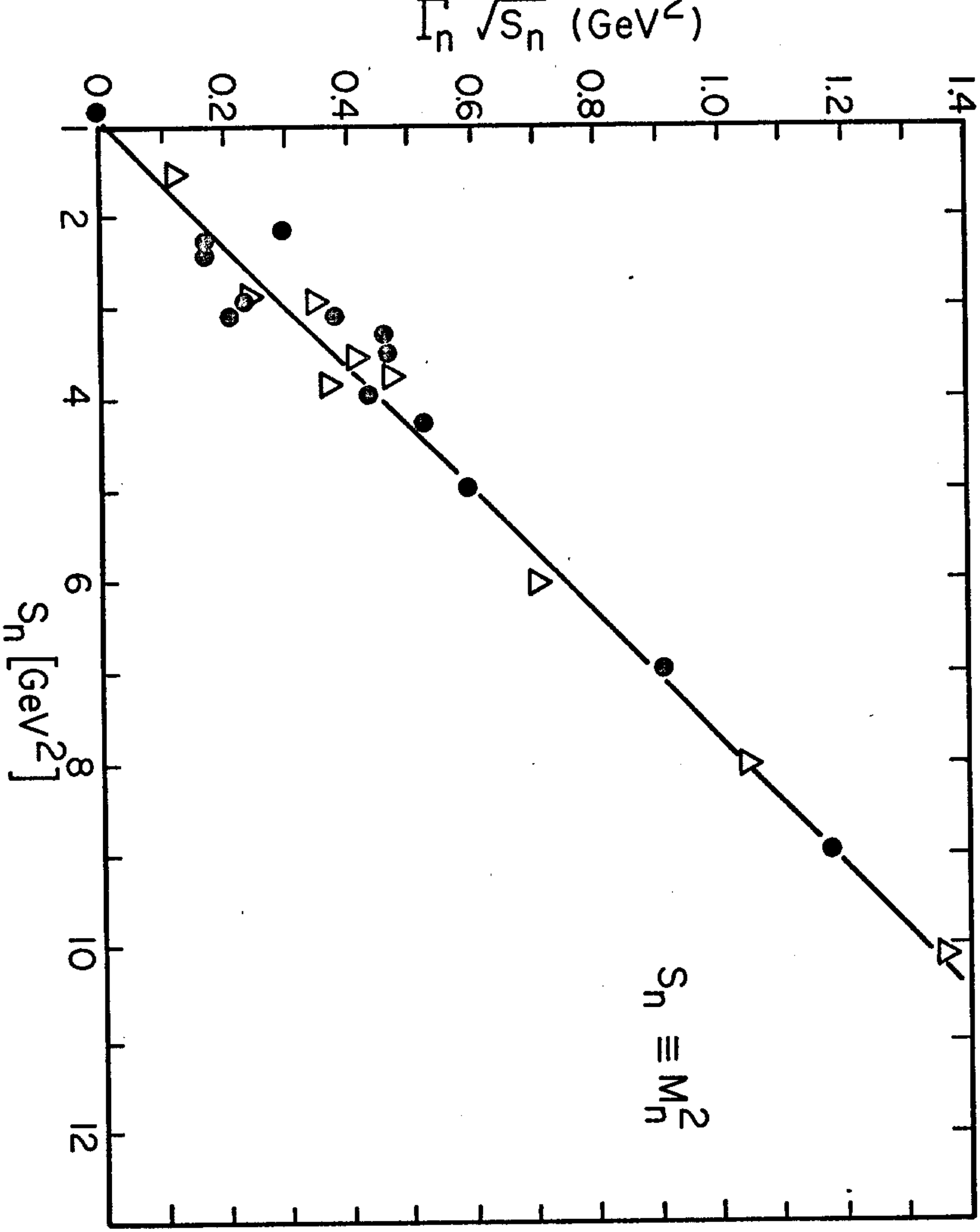
Figure 8. Ratio of differential cross sections of resonance electroproduction to elastic scattering. Data from Ref. 9.

figure1



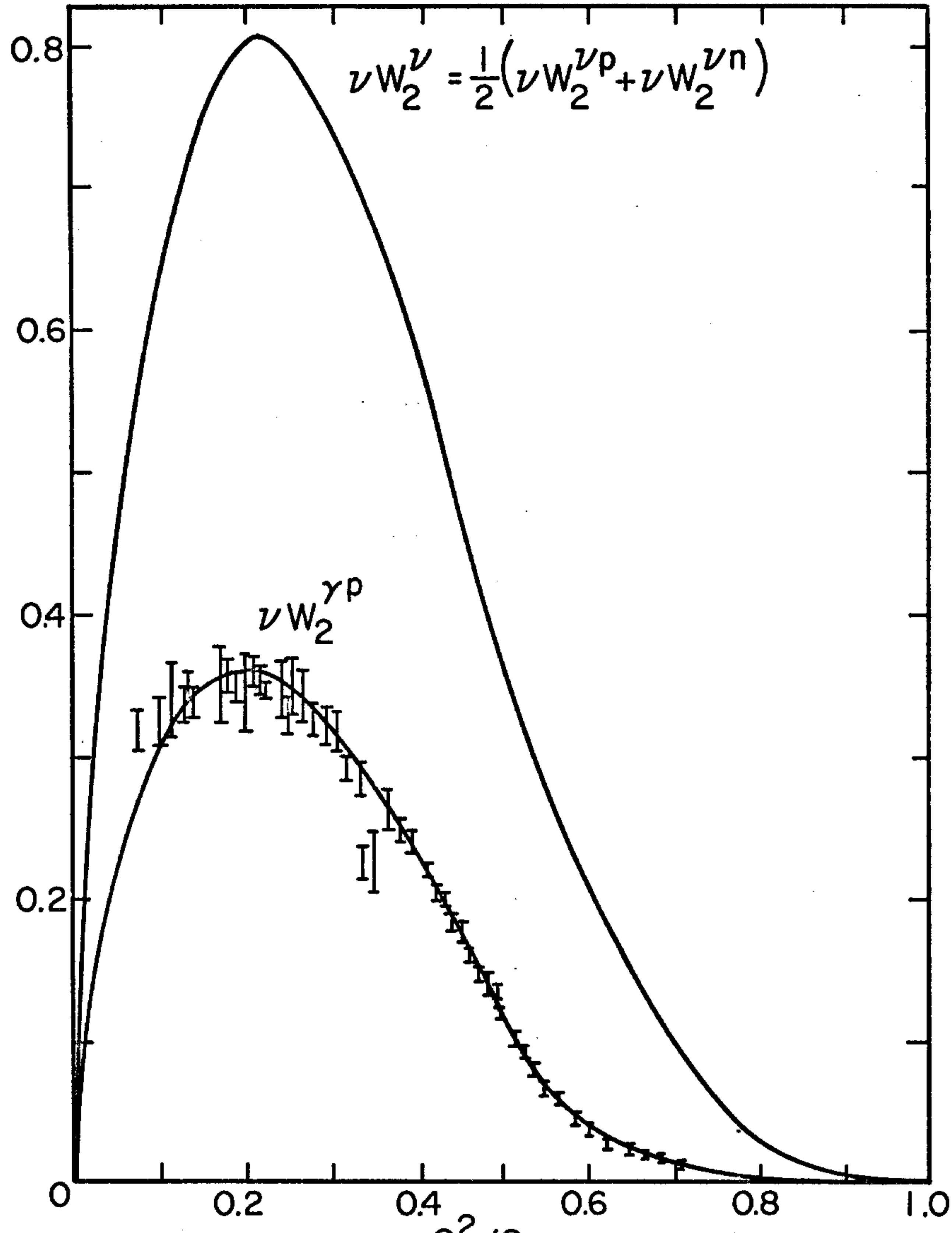
(νW_2)(Arbitrary Units)
0
1
2
3
$_6C^{12}$
2
4
6
8
10
12
14
$10^{-4}\,\frac{\nu}{q^2}\,[\mathrm{MeV}^{-1}]$
E = 65 MeV
Θ_{lab} = 180°
ν = E − E'
$q^2 = 4EE' \sin^2 \frac{\Theta_{lab}}{2}$
Average

figure 2



$\Gamma_n \sqrt{S_n}$ (GeV^2)
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
2
4
6
8
10
12
S_n [GeV^2]
$S_n \equiv M_n^2$

figure 3



$\nu W_2^{\nu} = \frac{1}{2}(\nu W_2^{\nu p} + \nu W_2^{\nu n})$
$\nu W_2^{\gamma p}$
0.8
0.6
0.4
0.2
0
0.2
0.4
0.6
0.8
1.0
$x = Q^2/2m\nu$

Figure 4

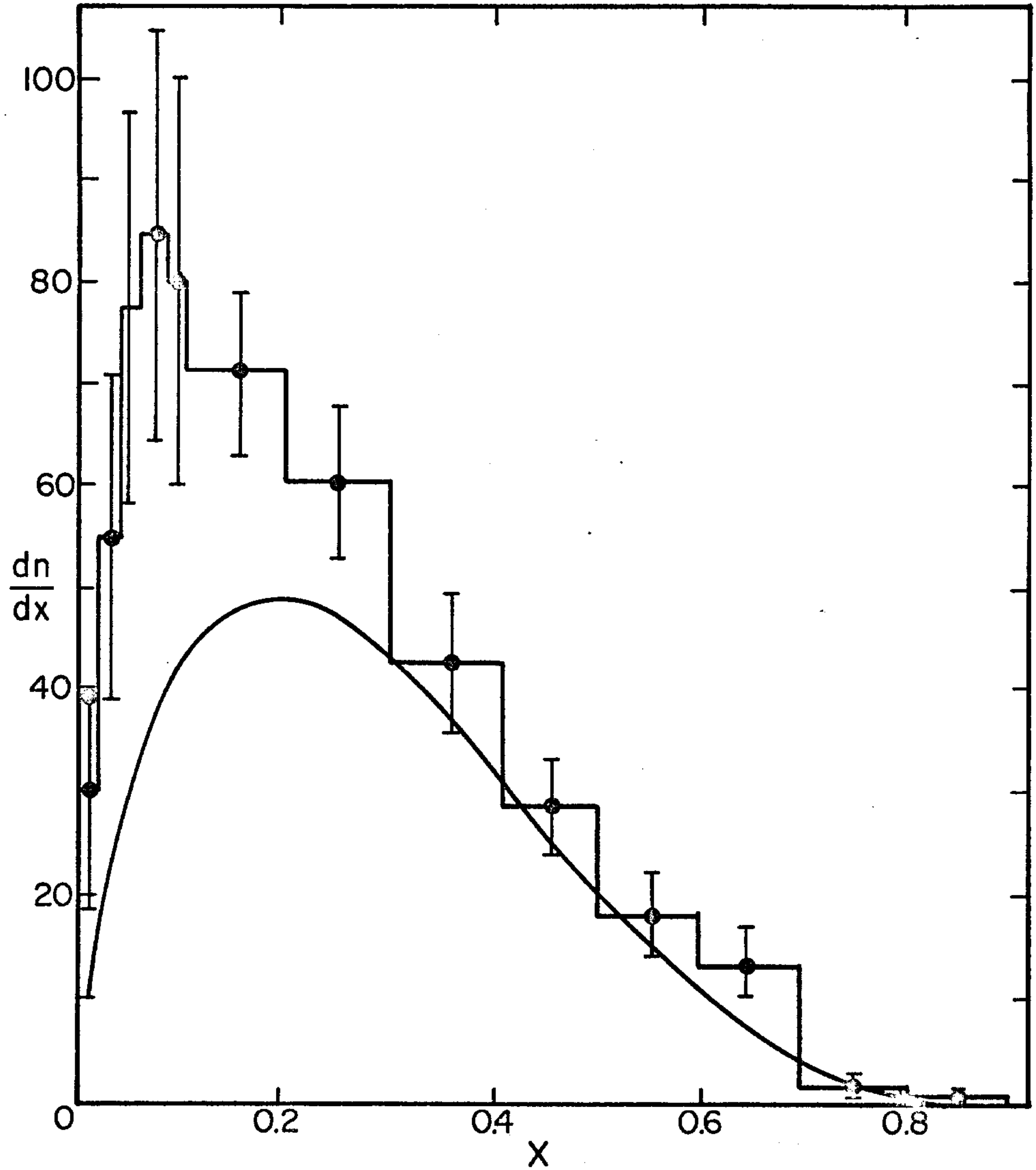


Figure 5

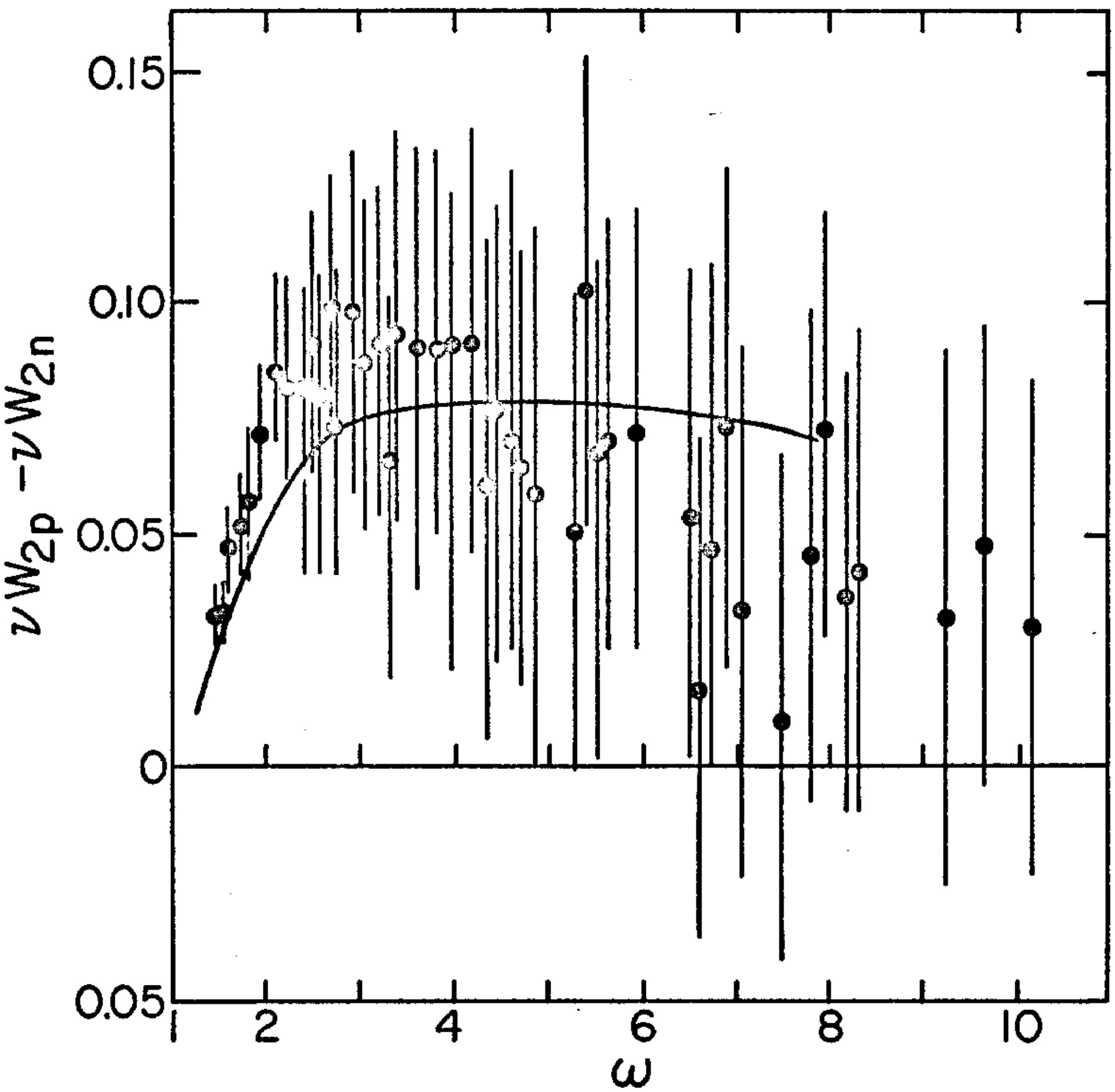


Figure 6

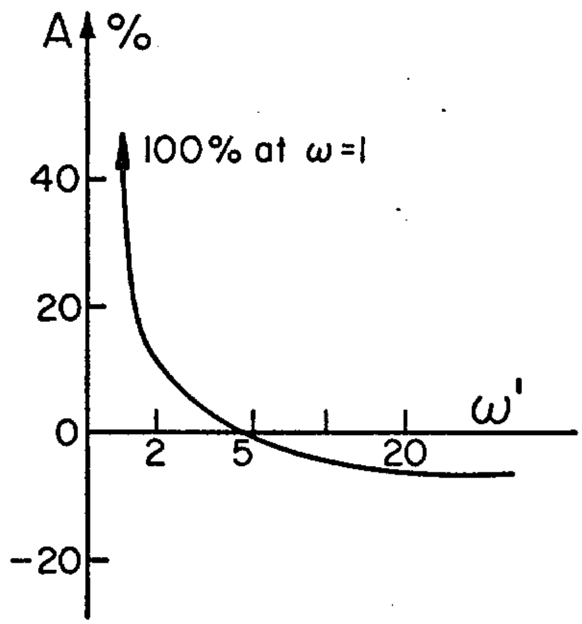


figure 7

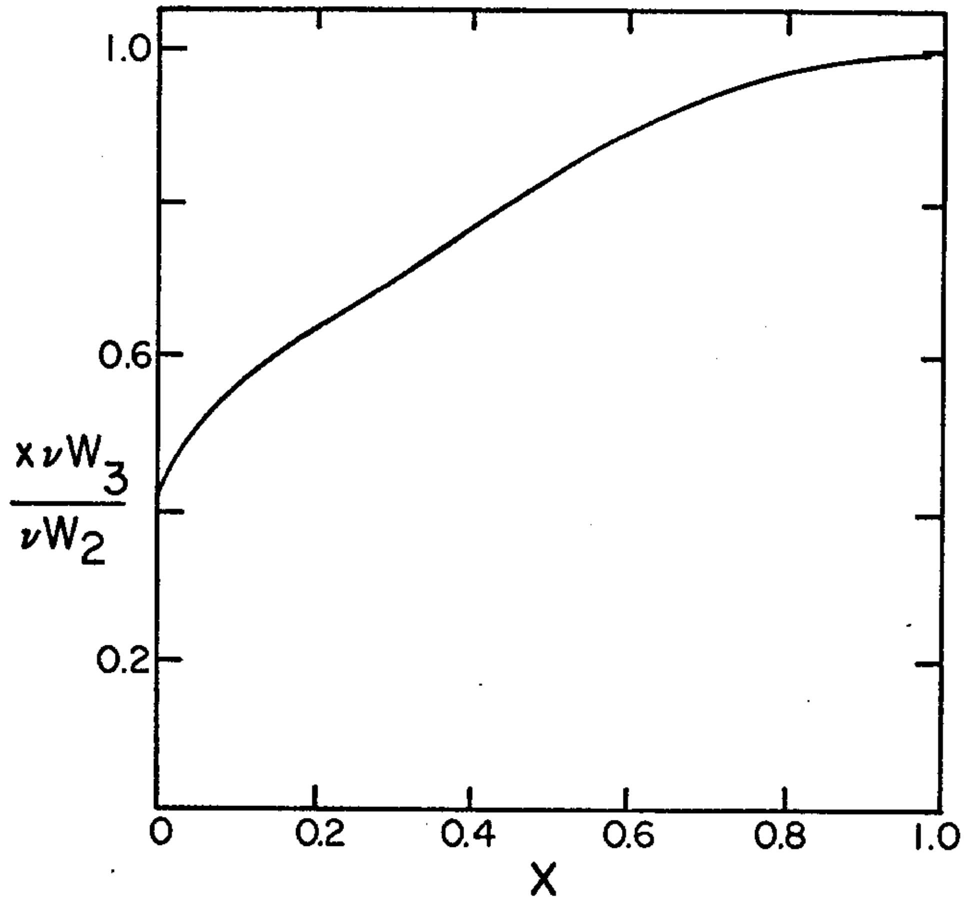


figure 8